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# Comparative Study of Different Vacation Schemes in an Inventory System with Production and Multiple Servers

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ABSTRACT. A MAP/PH(1), PH(2)/2 inventory model of multiple servers with production, in which each server takes multiple vacations, and these vacations are subject to different vacation scheduling regulations considered. The model is analyzed under different vacation scheduling tactics and a comparative study of different vacation schedules is undertaken.

## 1. INTRODUCTION

In typical production inventory systems, servers enter vacation mode either after serving every customer in the queue or after serving only a certain percentage of customers in the queue. However, with different vacation policies, the servers can decide whether or not to go on a vacation based on whether there are customers in the system with positive inventory levels after completing service. The servers go on vacation when each customer in the primary queue is finished serving (1-limited service discipline), or the server can take a vacation for a random duration (non-exhaustive), or the server opts to take a vacation after serving all customers accumulated during the vacation and those who arrived during the service process until the queue is exhausted (exhaustive service policy), or after completing service, with a positive inventory level, servers can decide whether to go on vacation or not if there are customers in the system (Bernoulli server vacation). The server may need to take breaks in the event of system malfunctions, equipment failures, testing, or repairs.The vacation queueing model research seems to have started in the 1970's. Significant survey papers on vacation models were proposed by Doshi [3] and Thegam [12]. Beena and Jose [2] detailed an inventory system of production comprising multiple servers, each of the servers is subject to multiple vacations, and the vacations are subject to the Bernoulli vacation policy. The initial study on queueing systems with one or more vacations was done by Levy and Yechiali [9]. Bernoulli vacation model studies were initiated by Keilson and Servi [6].

Suganya and Sivakumar [11] studied an inventory system in which numerous system characteristics were estimated, including joint probability distribution of the number of customers and the expected inventory level in the steady state. An M/M/N queueing system with Bernoulli vacation service strategy is studied by Krishnakumar and Madheswari. [7]. Krishnakumar et al. [8] examined a MAP/Ph(1),Ph(2)/2 queue with multiple vacations under the Bernoulli vacation scheduling service. Ayyappan and Gouthami [1] analyzed a queueing model with reneging of customers, that involved phase-type distributed service time and Bernoulli's scheduled vacation policy for the heterogeneous servers. Yue and Qin [13] analyzed a production inventory system with production time and production vacation times exponentially distributed and service time positive. Jose

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and Salini [5] examined the efficiency of two production inventory systems that had various rates of production. They were able to determine the optimum value of the coefficient of replenishment rate that would minimize the total expected cost. Jose and Beena [4] analyzed a multi-server production inventory system with the retrial of customers.
Section 2 of the next section is focused on the description of the model. The analysis and the model stability are provided in sections 3 and 4. Sections 5 and 6 cover steady-state analysis and performance measures. Sections 7 and 8 are concerned with correlation analysis and numerical experiments respectively. Section 9 concludes the discussion.

## 2. Description of the Model

A production inventory system with two heterogeneous servers that offers different rates of services to customers, and only allows one client to use the system at a time is considered. The arrival of customers are according to $MAP$ with representation $(D_0, D_1)_l$ and the stationary arrival rate is given by $\lambda = \sigma D_1 e$, where $\sigma$ be the stationary probability vectors of $(D_0 + D_1)$ of length $l$. Service rates of server 1 and server 2 are phase type distributed having representations $(\alpha, S)_m$ and $(\beta, T)_n$ respectively. $S$ is a matrix of order $m$ and $\alpha e = 1$; $S^0$ is a column vector such that $Se + S^0 = 0$. Similarly $T$ is a matrix of order $n$ and $\beta e = 1$; $T^0$ is a column vector such that $Te + T^0 = 0$. The mean service time for server 1 and 2 are given by $\mu_1 = \alpha S^{-1} e_m$ and $\mu_2 = \beta T^{-1} e_n$ respectively. The servers enter into a vacation mode when the system runs out of stock or no customers are present in the system or the level of inventory and customers are zero. Vacation duration of servers 1 and 2 are assumed to be independent and identically distributed as exponential with parameters $\theta_1$ and $\theta_2$. If the system has no customers, the inventory level is zero or both, the servers will always take a vacation. After the service is over, if there are clients in the waiting area and a positive inventory level, the servers can choose to continue serving customers with its complimentary probability, $q_i = 1 - p_i, i = 1, 2$, or take a vacation with probability $p_i, i = 1, 2$. After a vacation, if the inventory is zero, the system is empty or both, the servers go back to their previous state. Until they detect that the system is not empty and has a positive inventory level, the servers will keep doing this. Produced items are accessible only after a certain period and are distributed according to an exponential distribution with parameter $\gamma (> 0)$. The notations and assumptions used in this model are

i) N(t) indicates the number of customers in the system at time $t$
ii) I(t) describes the level of stock at time $t$
iii) C(t) indicates the status of servers 1 and 2
iv) F(t) denotes the production status
v) $J_0(t)$ indicates the phase of the arrival process
vi) $J_1(t)$ and $J_2(t)$ indicate the phases of the service processes of servers 1 and 2
vii) e=(1,1,1,...,1)′, column vector of appropriate dimension containing all ones
viii) $\rtimes_1 = l(S - s) + lS$, $\rtimes_2 = lm(S - s) + lm(S - 1)$
$\rtimes_3 = ln(S - s) + ln(S - 1)$, $\rtimes_4 = lmn(S - s) + lmn(S - 2)$,
$e_{\rtimes_1 + \rtimes_2 + \rtimes_3}(\rtimes_1)$ is a $(\rtimes_1 + \rtimes_2 + \rtimes_3) \times 1$ column vector with first $\rtimes_1$ elements are 1 and all other entries are zero.
$e_{\rtimes_1 + \rtimes_2 + \rtimes_3 + \rtimes_4}(\rtimes_1)$ is a $(\rtimes_1 + \rtimes_2 + \rtimes_3 + \rtimes_4) \times 1$ column vector with first $\rtimes_1$ elements are 1 and all other entries are zeros.
$e_{\rtimes_1 + \rtimes_2 + \rtimes_3 + \rtimes_4}(\rtimes_2)$ is a $(\rtimes_1 + \rtimes_2 + \rtimes_3 + \rtimes_4) \times 1$ column vector with $\rtimes_1 + 1$ to $\rtimes_2$ elements are 1 and all other entries are zeros.
$e_{\rtimes_1 + \rtimes_2 + \rtimes_3 + \rtimes_4}(\rtimes_3)$ is a $(\rtimes_1 + \rtimes_2 + \rtimes_3 + \rtimes_4) \times 1$ column vector with $\rtimes_1 + \rtimes_2 + 1$ to $\rtimes_3$ elements are 1 and all other entries are zeros.

$e_{\rtimes_1+\rtimes_2+\rtimes_3+\rtimes_4}(\rtimes_4)$ is a $(\rtimes_1+\rtimes_2+\rtimes_3+\rtimes_4)\times 1$ column vector with $\rtimes_1+\rtimes_2+\rtimes_3+1$ to $\rtimes_4$ elements are 1 and all other entries are zeros.

## 3. Analysis

The stochastic process $\{X(t) = (N(t), C(t), F(t), I(t), J_0(t), J_1(t), J_2(t)), t \geq 0\}$ is a level independent quasi birth death process (LIQBD) on the state space

$$\Omega = \bigcup_{0}^{\infty} \bar{\boldsymbol{z}}$$

where,

$$\bar{\mathbf{0}} = \Big\{(0,0,0,j,j_0),(s+1 \leq j \leq S) \bigcup (0,0,1,j,j_0),(0 \leq j \leq S-1)$$

$$\bar{\mathbf{1}} = \begin{cases} (1,0,0,j,j_0),(s+1 \leq j \leq S) \bigcup (1,0,1,j,j_0),(0 \leq j \leq S-1) \\ \bigcup (1,1,0,j,j_0,j_1),(s+1 \leq j \leq S) \bigcup (1,1,1,j,j_0,j_1),(1 \leq j \leq S-1) \\ \bigcup (1,2,0,j,j_0,j_2),(s+1 \leq j \leq S) \bigcup (1,2,1,j,j_0,j_2),(1 \leq j \leq S-1) \end{cases}$$

and for $\boldsymbol{z} \geq 2$

$$\bar{\boldsymbol{z}} = \begin{cases} (z,0,0,j,j_0),(s+1 \leq j \leq S) \bigcup (z,0,1,j,j_0),(0 \leq j \leq S-1) \\ \bigcup (z,1,0,j,j_0,j_1),(s+1 \leq j \leq S) \bigcup (z,1,1,j,j_0,j_1),(1 \leq j \leq S-1) \\ \bigcup (z,2,0,j,j_0,j_2),(s+1 \leq j \leq S) \bigcup (z,2,1,j,j_0,j_2),(1 \leq j \leq S-1) \\ \bigcup (z,3,0,j,j_0,j_1,j_2),(s+1 \leq j \leq S) \bigcup (z,3,1,j,j_0,j_1,j_2),(2 \leq j \leq S-1) \end{cases}$$

The following events can cause changes in the Markov chain: new customers joining the system, services finished, the production of an item occurring, the servers' vacation comes to an end, and transitions that retain the first four coordinates of the state space. The generator matrix of the Markov process can be written as

$$Q = \begin{bmatrix} C_{00} & C_{01} & 0 & 0 & 0 & \dots \\ C_{10} & C_{11} & C_{12} & 0 & 0 & \dots \\ 0 & C_{21} & C_1 & C_0 & 0 & \dots \\ 0 & 0 & C_2 & C_1 & C_0 & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

where $C_0, C_1, C_2$ are square matrices of order
$l(2S-s) + lm(2S-s-1) + ln(2S-s-1) + lmn(2S-s-2)$.

## 4. Stability condition and computation of steady state probability vector

To derive the system stability, define a matrix $\tilde{C} = C_0 + C_1 + C_2$.
$\tilde{C}$ is irreducible so there exist a $1 \times (\rtimes_1 + \rtimes_2 + \rtimes_3 + \rtimes_4)$ stationary probability vector $\Pi$ satisfying $\Pi\tilde{C} = 0$ and $\Pi e = 1$. $\Pi$ can be partitioned as
$\Pi = (\pi^{[i]}, i = 0,1,2,3)$ where each $\pi^{[i]} = \{(\pi^{[i,0]}, \pi^{[i,1]}), i = 0,1,2,3\}$

$$\pi^{[i,0]} = \begin{cases} (\pi^{[i,0,s+1,l]}, \dots, \pi^{[i,0,S,l]}), i = 0 \\ (\pi^{[i,0,s+1,l,m]}, \dots, \pi^{[i,0,S,l,m]}), i = 1 \\ (\pi^{[i,0,s+1,l,n]}, \dots, \pi^{[i,0,S,l,n]}), i = 2 \\ (\pi^{[i,0,s+1,l,m,n]}, \dots, \pi^{[i,0,S,l,m,n]}), i = 3 \end{cases}$$

$$\pi^{[i,1]}=\begin{cases}(\pi^{[i,1,0,l]},\dots,\pi^{[i,1,S-1,l]}), i=0\\(\pi^{[i,1,1,l,m]},\dots,\pi^{[i,1,S-1,l,m]}), i=1\\(\pi^{[i,1,1,l,n]},\dots,\pi^{[i,1,S-1,l,n]}), i=2\\(\pi^{[i,1,2,l,m,n]},\dots,\pi^{[i,1,S-1,l,m,n]}), i=3\end{cases}$$

Based on the the renowned result about the standard drift condition of Nuets [10], $\Pi C_0 e < \Pi C_2 e$ is necessary and sufficient condition for the QBD process to be stable.

$$\Pi C_0 e=\begin{cases}\pi^{[0,0]}[D_1 I_{S-s}]e_{l(S-s)}+\pi^{[0,1]}[D_1 I_S]e_{lS}+\pi^{[1,0]}[D_1 I_{S-s}e_{l(S-s)}]\otimes e_m\\+\pi^{[1,1]}[D_1 I_{S-1}e_{l(S-1)}]\otimes e_m+\pi^{[2,0]}[D_1 I_{S-s}e_{l(S-s)}]\otimes e_n\\+\pi^{[2,1]}[D_1 I_{S-1}e_{l(S-1)}]\otimes e_n+\pi^{[3,0]}[D_1 I_{S-s}e_{l(S-s)}]\otimes e_{mn}\\+\pi^{[3,1]}[D_1 I_{S-2}e_{l(S-2)}]\otimes e_{mn}\end{cases}$$

$$\Pi C_2 e=\begin{cases}\pi^{[1,0]}[(e_l\otimes S^0)\otimes e_{S-s}]+\Big[\pi^{[1,1,1]}[e_l\otimes p_1 S^0]+\pi^{[1,1,2]}[e_l\otimes S^0]\\+\dots\dots\dots+\pi^{[1,1,S-1]}[e_l\otimes S^0]\Big]+\pi^{[2,0]}[(e_l\otimes T^0)\otimes e_{S-s}]\\+\Big[\pi^{[2,1,1]}[e_l\otimes p_2 T^0]+\pi^{[2,1,2]}[e_l\otimes T^0]+\cdots+\pi^{[2,1,S-1]}[e_l\otimes T^0]\Big]\\+\pi^{[3,0]}[e_l\otimes(S^0\oplus T^0)\otimes e_{S-s}]+\Big[\pi^{[3,1,2]}[e_l\otimes(p_1 S^0\oplus p_2 T^0)]\\+\pi^{[3,1,3]}[e_l\otimes(S^0\oplus T^0)]+\dots\dots+\pi^{[3,1,S-1]}[e_l\otimes(S^0\oplus T^0)]\Big]\end{cases}$$

## 5. Steady State Analysis

Under the stability condition of the system, there exist a steady state probability vector X=$(X_0, X_1,\dots)$, satisfying $XQ=0, Xe=1$ and it can be partitioned as

$$X_0=(y_{0,0,0,s+1},\dots y_{0,0,0,S},y_{0,0,1,0},\dots y_{0,0,1,S-1})$$

$$X_1=\begin{cases}(y_{1,0,0,s+1},\dots,y_{1,0,0,S},y_{1,0,1,0}\dots,y_{1,0,1,S-1},y_{1,1,0,s+1}\dots,y_{1,1,0,S})\\(y_{1,1,1,1}\dots y_{1,1,1,S-1},y_{1,2,0,s+1}\dots y_{1,2,0,S},y_{1,2,1,1}\dots\dots y_{1,2,1,S-1})\end{cases}$$

and for $i\geq 2$

$$X_i=\begin{cases}(y_{i,0,0,s+1},\dots,y_{i,0,0,S},y_{i,0,1,0}\dots,y_{i,0,1,S-1},y_{i,1,0,s+1}\dots,y_{i,1,0,S})\\(y_{i,1,1,1}\dots y_{i,1,1,S-1},y_{i,2,0,s+1}\dots y_{i,2,0,S},y_{i,2,1,1}\dots\dots y_{i,2,1,S-1})\\y_{i,3,0,s+1},\dots,y_{i,3,0,S},y_{i,3,1,2}\dots\dots y_{i,3,1,S-1}\end{cases}$$

where

$$y_{i,0,k,j}=\begin{cases}(y_{i,0,0,j,1}\dots\dots y_{i,0,0,j,m_0}), s+1\leq j\leq S, i\geq 0,\\(y_{i,0,1,j,1}\dots\dots y_{i,0,1,j,m_0}), 0\leq j\leq S-1, i\geq 0,\end{cases}$$

$$y_{i,1,k,j}=\begin{cases}(y_{i,1,0,j,1,1}\dots\dots y_{i,1,0,j,m_0,m_1}), s+1\leq j\leq S, i\geq 1\\(y_{i,1,1,j,1,1}\dots\dots y_{i,1,1,j,m_0,m_1}), 1\leq j\leq S-1, i\geq 1\end{cases}$$

$$y_{i,2,k,j}=\begin{cases}(y_{i,2,0,j,1,1}\dots\dots y_{i,2,0,j,m_0,m_2}), s+1\leq j\leq S, i\geq 1\\(y_{i,2,1,j,1,1}\dots\dots y_{i,2,1,j,m_0,m_2}), 1\leq j\leq S-1, i\geq 1\end{cases}$$

$$y_{i,3,k,j}=\begin{cases}(y_{i,3,0,j,1,1,1}\dots\dots y_{i,3,0,j,m_0,m_1,m_2}), s+1\leq j\leq S, i\geq 2\\(y_{i,3,1,j,1,1,1}\dots\dots y_{i,3,1,j,m_0,m_1,m_2}), 2\leq j\leq S-1, i\geq 2\end{cases}$$

One can obtain the sub vectors of $X$ by solving

$$X_0 C_{00}+X_1 C_{10}=0 \tag{5.1}$$

(5.2) $$X_0C_{01} + X_1C_{11} + X_2C_{21} = 0$$

(5.3) $$X_1C_{12} + X_2[C_1 + RC_2] = 0$$

(5.4) $$X_i = X_{i-1} * R,\ i = 3, 4, 5 \dots$$

The normalizing equation is

(5.5) $$X_0e + X_1e_{(\rtimes_1+\rtimes_2+\rtimes_3)} + X_2(I - R)^{-1}e_{(\rtimes_1+\rtimes_2+\rtimes_3+\rtimes_4)} = 1$$

The rate matrix $R$ can be obtained from $R = -C_0(C_1)^{-1} - R^2C_2(C_1)^{-1}$ and approximated by the successive substitution method developed by Neuts. [10]. If $R$ is calculated then the sub vectors $X_0$, $X_1$, and $X_2$ and $X_i, i \geq 3$ can be calculated using equations 5.1,5.2,5.3,5.4 and 5.5.

## 6. Performance measures

(i) Expected number of customers in the system :

$$\aleph_{EC} = X_1e_{(\rtimes_1+\rtimes_2+\rtimes_3)} + X_2[2(I - R)^{-1} + R(I - R)^{-2}]\boldsymbol{e}_{(\rtimes_1+\rtimes_2+\rtimes_3+\rtimes_4)}$$

(ii) Mean number of customers in the system when both the servers are on vacation:

$$\aleph_{ECV} = X_1\boldsymbol{e}_{(\rtimes_1+\rtimes_2+\rtimes_3}(\rtimes_1) + X_2[2(I - R)^{-1} + R(I - R)^{-2}]\boldsymbol{e}_{(\rtimes_1+\rtimes_2+\rtimes_3+\rtimes_4)}(\rtimes_1)$$

(iii) Expected switching rate:

$$\aleph_{SWR} = \sum_{i=1}^{\infty} y_{(i,1,0,s+1)}[I_l \otimes S^0]\boldsymbol{e}$$
$$+\sum_{i=1}^{\infty} y_{(i,2,0,s+1)}[I_l \otimes T^0]\boldsymbol{e} + \sum_{i=2}^{\infty} y_{(i,3,0,s+1)}[I_l \otimes (S^0 \otimes T^0)]\boldsymbol{e}$$

(iv) Expected inventory level:

$$\aleph_{EI} = \sum_{i=0}^{\infty}\sum_{j=1}^{S-1}\sum_{j_0=1}^{l}\left[jy_{(i,0,1,j,j_0)} + \sum_{j_1=1}^{m} jy_{(i,1,1,j,j_0,j_1)} + \sum_{j_2=1}^{n} jy_{(i,2,1,j,j_0,j_2)}\right]$$

$$\sum_{i=0}^{\infty}\sum_{j=s+1}^{S}\sum_{j_0=1}^{l}\left[jy_{(i,0,0,j,j_0)} + \sum_{j_1=1}^{m} jy_{(i,1,0,j,j_0,j_1)} + \sum_{j_2=1}^{n} jy_{(i,2,0,j,j_0,j_2)}\right]$$

$$\sum_{i=2}^{\infty}\sum_{j=s+1}^{S}\sum_{j_0=1}^{l}\sum_{j_1=1}^{m}\sum_{j_2=1}^{n}\left[jy_{(i,3,0,j,j_0,j_1,j_2)} + jy_{(i,3,1,j,j_0,j_1,j_2)}\right]$$

(v) Expected inventory level when there are active servers:

$$\aleph_{EIA} = \sum_{i=2}^{\infty}\sum_{j_0=1}^{l}\sum_{j_1=1}^{m}\sum_{j_2=1}^{n}\left[\sum_{j=s+1}^{S} jy_{(i,3,0,j,j_0,j_1,j_2)} + \sum_{j=2}^{S-1} jy_{(i,3,1,j,j_0,j_1,j_2)}\right]$$

(vi) Expected inventory level when servers are on vacation:

$$\aleph_{EIB} = \sum_{i=0}^{\infty}\sum_{j_0=1}^{l}\left[\sum_{j=s+1}^{S} jy_{(i,0,0,j,j_0)} + \sum_{j=1}^{S-1} jy_{(i,0,1,j,j_0)}\right]$$

(vii) Expected number of departures after completing service:

$$\aleph_{EDS} = \sum_{i=1}^{\infty} \sum_{j=s+1}^{S} \left[ y_{i,1,0,j}(I_l \otimes S^0)e + y_{i,2,0,j}(I_l \otimes T^0)e \right]$$

$$+ \sum_{i=1}^{\infty} \sum_{j=1}^{S-1} \left[ y_{i,1,1,j}(I_l \otimes S^0)e + y_{i,2,1,j}(I_l \otimes T^0)e \right]$$

$$+ \sum_{i=2}^{\infty} \left[ \sum_{j=s+1}^{S} y_{i,3,0,j}(I_l \otimes (S^0 \oplus T^0))e + \sum_{j=2}^{S-1} y_{i,3,1,j}(I_l \otimes (S^0 \oplus T^0))e \right]$$

(viii) Expected number of customers in the system while servers are in service:
$\aleph_{ECA} = X_2 e_{(\rtimes_1+\rtimes_2+\rtimes_3+\rtimes_4)}(\rtimes_4)$

(ix) Expected number of customers in the system when server 1 is functioning and server 2 is on vacation:

$$\aleph_{EA1} = X_1 e_{(\rtimes_1+\rtimes_2+\rtimes_3)}(\rtimes_2) + X_2[2(I-R)^{-1} + R(I-R)^{-2}]\boldsymbol{e}_{(\rtimes_1+\rtimes_2+\rtimes_3+\rtimes_4)}(\rtimes_2)$$

(x) Expected number of customers in the system when server 2 is functioning and server 1 is on vacation:

$$\aleph_{EA2} = X_1 e_{(\rtimes_1+\rtimes_2+\rtimes_3)}(\rtimes_3) + X_2[2(I-R)^{-1} + R(I-R)^{-2}]\boldsymbol{e}_{(\rtimes_1+\rtimes_2+\rtimes_3+\rtimes_4)}(\rtimes_3)$$

## 7. Correlation Analysis

The method used for estimating the system's expected total cost ($T_{COST}$) per unit per unit time is

$$T_{COST} = (C + (S-s)c_1)\aleph_{SWR} + c_2\aleph_{EI} + c_3\aleph_{EC} + c_4\aleph_{EDS}$$

where, $C$= fixed cost per unit per unit time $c_1$= running cost of production unit per unit per unit time, $c_2$= holding cost of inventory per unit per unit time, $c_3$= holding cost of customers per unit per unit time, $c_4$= cost due to service per unit per unit time.

## 8. Numerical Illustrations

The arrival processes marked as $MAP^-$ and $MAP^+$ respectively, exhibit covariance of 2.0239 and negative and positive correlations of -0.4791 and 0.4791. Select the values of parameters as $\alpha = \begin{bmatrix} 0.3 & 0.5 & 0.2 \end{bmatrix}$,

$\beta = \begin{bmatrix} 0.3 & 0.4 & 0.3 \end{bmatrix}$, $S = \begin{bmatrix} -6 & 3 & 0 \\ 1 & -4 & 2 \\ 2 & 0 & -5 \end{bmatrix}$, $T = \begin{bmatrix} -3 & 2 & 0 \\ 0 & -5 & 2 \\ 1 & 0 & -3 \end{bmatrix}$,

$S^0 = \begin{bmatrix} 3 \\ 1 \\ 3 \end{bmatrix}$, $T^0 = \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}$

1.Map with negative correlation $MAP^-$:

$D_0 = \begin{bmatrix} -3 & 3 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -450.5 \end{bmatrix}$, $D_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0.02 & 0 & 1.98 \\ 445.995 & 0 & 4.505 \end{bmatrix}$

2.Map with positive correlation $MAP^{+}$:

$$D_0 = \begin{bmatrix} -3 & 3 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -450.5 \end{bmatrix}, D_1 = \begin{bmatrix} 0 & 0 & 0 \\ 1.98 & 0 & 0.02 \\ 4.505 & 0 & 445.995 \end{bmatrix}$$

For the positively and negatively correlated arrivals, we take into consideration the long-term expected cost pattern based on variations in the values of key parameters in different service distributions. The impact of both positive and negative correlated inter-arrival time on performance measures and expected costs are shown in tables 1, 2, and 3 for the Bernoulli vacation scheduling service, the exhaustive service policy, and the 1-limited service policy over a range of $S$ values, with the other parameters fixed. Choose $C = 10, c_1 = 2, c_2 = 50, c_3 = 10, c_4 = 45, \theta_1 = 2, \theta_2 = 1, \gamma = 3.2$

TABLE 1. Effect of the values of $S$ (BER vacation scheduling service ($p_1 = 0.8, p_2 = 0.7, s = 5$))

| | | | | $MAP^{-}$ | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| S | $\aleph_{EC}$ | $\aleph_{ECA}$ | $\aleph_{ECV}$ | $\aleph_{SWR}$ | $\aleph_{EI}$ | $\aleph_{EIA}$ | $\aleph_{EIB}$ | $\aleph_{EDS}$ | $T_{COST}$ |
| 10 | 0.1693 | 0.0055 | 0.0245 | 0.0102 | 1.3680 | 0.1589 | 0.5804 | 0.0942 | 71.8127 |
| 11 | 0.1737 | 0.0055 | 0.0245 | 0.0119 | 1.3384 | 0.1567 | 0.5700 | 0.0994 | 70.4820 |
| 12 | 0.1785 | 0.0057 | 0.0245 | 0.0141 | 1.3090 | 0.1550 | 0.5594 | 0.1057 | 69.1870 |
| 13 | 0.1834 | 0.0058 | 0.0242 | 0.0170 | 1.2788 | 0.1537 | 0.5481 | 0.1134 | 67.8833 |
| 14 | 0.1880 | 0.0058 | 0.0234 | 0.0217 | 1.2462 | 0.1530 | 0.5357 | 0.1229 | 66.5081 |
| 15 | 0.1915 | 0.0058 | 0.0219 | 0.0229 | 1.2080 | 0.1526 | 0.5212 | 0.1348 | 64.7962 |
| | | | | $MAP^{+}$ | | | | | |
| 10 | 3.0301 | 0.0393 | 0.5346 | 0.0127 | 2.7122 | 0.9575 | 0.4313 | 0.3223 | 171.1259 |
| 11 | 3.1449 | 0.0412 | 0.5513 | 0.0155 | 2.5937 | 0.9223 | 0.4012 | 0.3365 | 166.6147 |
| 12 | 3.2746 | 0.0451 | 0.5686 | 0.0193 | 2.4729 | 0.8874 | 0.3694 | 0.3534 | 162.1907 |
| 13 | 3.4205 | 0.0482 | 0.5854 | 0.0249 | 2.3485 | 0.8527 | 0.3356 | 0.3738 | 157.8318 |
| 14 | 3.5846 | 0.0499 | 0.5996 | 0.0332 | 2.2191 | 0.8186 | 0.2993 | 0.3990 | 153.5141 |
| 15 | 3.7699 | 0.0523 | 0.6067 | 0.0321 | 2.0836 | 0.7864 | 0.2602 | 0.4317 | 148.9943 |

TABLE 2. Effect of the values of $S$ (Exhaustive service policy $(s = 2)$)

| S | $\aleph_{EC}$ | $\aleph_{ECA}$ | $\aleph_{ECV}$ | $\aleph_{SWR}$ | $\aleph_{EI}$ | $\aleph_{EIA}$ | $\aleph_{EIB}$ | $\aleph_{EDS}$ | $T_{COST}$ |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | $MAP^-$ | | | | |
| 10 | 0.1462 | 0.0022 | 0.0122 | 0.0112 | 0.9712 | 0.0671 | 0.4694 | 0.0931 | 51.7116 |
| 11 | 0.1426 | 0.0022 | 0.0119 | 0.0091 | 1.0224 | 0.0726 | 0.4835 | 0.0910 | 54.1677 |
| 12 | 0.1403 | 0.0021 | 0.0117 | 0.0075 | 1.0786 | 0.0786 | 0.4987 | 0.0897 | 56.9028 |
| 13 | 0.1390 | 0.0021 | 0.0116 | 0.0063 | 1.1400 | 0.0851 | 0.5152 | 0.0890 | 59.9259 |
| 14 | 0.1386 | 0.0020 | 0.0116 | 0.0054 | 1.2076 | 0.0921 | 0.5332 | 0.0889 | 63.2840 |
| 15 | 0.1391 | 0.0020 | 0.0116 | 0.0047 | 1.2821 | 0.0998 | 0.5529 | 0.0893 | 67.0037 |
| | | | | | $MAP^+$ | | | | |
| 10 | 1.6937 | 0.0001 | 0.0058 | 0.0084 | 1.3777 | 0.9584 | 0.1954 | 0.5923 | 94.9246 |
| 11 | 1.6937 | 0.0001 | 0.0058 | 0.0084 | 1.3777 | 0.9584 | 0.1954 | 0.5923 | 94.9246 |
| 12 | 1.6921 | 0.0029 | 0.0059 | 0.0022 | 1.5124 | 1.0587 | 0.2062 | 0.5822 | 101.3373 |
| 13 | 1.6742 | 0.0000 | 0.0058 | 0.0039 | 1.6511 | 1.1689 | 0.2136 | 0.5808 | 108.1272 |
| 14 | 1.6721 | 0.0010 | 0.0058 | 0.0025 | 1.7928 | 1.2784 | 0.2221 | 0.5780 | 115.1117 |
| 15 | 1.6726 | 0.0003 | 0.0058 | 0.0023 | 1.9404 | 1.3934 | 0.2299 | 0.5790 | 122.5101 |

TABLE 3. Effect of values of $S$ (1-Limited service policy $(s = 5)$)

| S | $\aleph_{EC}$ | $\aleph_{ECA}$ | $\aleph_{ECV}$ | $\aleph_{SWR}$ | $\aleph_{EI}$ | $\aleph_{EIA}$ | $\aleph_{EIB}$ | $\aleph_{EDS}$ | $T_{COST}$ |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | $MAP^-$ | | | | |
| 10 | 0.2375 | 0.0090 | 0.0347 | 0.0311 | 1.3092 | 0.1692 | 0.5494 | 0.1385 | 70.5330 |
| 11 | 0.2380 | 0.0091 | 0.0383 | 0.0290 | 1.3476 | 0.1678 | 0.5651 | 0.1242 | 72.2618 |
| 12 | 0.2347 | 0.0094 | 0.0403 | 0.0236 | 1.3734 | 0.1660 | 0.5767 | 0.1124 | 73.2707 |
| 13 | 0.2295 | 0.0093 | 0.0412 | 0.0201 | 1.3912 | 0.1640 | 0.5856 | 0.1026 | 73.9139 |
| 14 | 0.2232 | 0.0090 | 0.0415 | 0.0172 | 1.4036 | 0.1620 | 0.5926 | 0.0943 | 74.3107 |
| 15 | 0.2165 | 0.0090 | 0.0413 | 0.0149 | 1.4125 | 0.1601 | 0.5982 | 0.0872 | 74.5458 |
| | | | | | $MAP^+$ | | | | |
| 10 | 3.7596 | 0.0938 | 0.8935 | 0.0580 | 1.8515 | 0.5169 | 0.2645 | 0.2647 | 135.2999 |
| 11 | 3.5843 | 0.0928 | 0.9015 | 0.0528 | 1.9272 | 0.5091 | 0.2966 | 0.2316 | 136.8405 |
| 12 | 3.4262 | 0.0941 | 0.8957 | 0.0438 | 1.9959 | 0.5046 | 0.3253 | 0.2072 | 138.2149 |
| 13 | 3.2800 | 0.0924 | 0.8822 | 0.0375 | 2.0575 | 0.5015 | 0.3514 | 0.1881 | 139.4727 |
| 14 | 3.1431 | 0.0894 | 0.8640 | 0.0326 | 2.1125 | 0.4990 | 0.3753 | 0.1726 | 140.5592 |
| 15 | 3.0140 | 0.0892 | 0.8430 | 0.0286 | 2.1614 | 0.4966 | 0.3974 | 0.1596 | 141.4644 |

It is evident from the table data that the lowest expected cost can be achieved by using MAP with a negative correlation.

## 9. CONCLUSION

This article presents the findings of an in-depth analysis of a production inventory model that offers servers a wide range of vacation options. This model has the advantage that it allows the servers to choose vacation strategies according to their requirements and preferences. By taking into account servers with more than two and the distribution of server vacation as a phase type, this study can be further expanded.

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