CREAT. MATH. INFORM.
Volume **28** (2019), No. 1,
Pages 77 - 83

Online version at `https://creative-mathematics.cunbm.utcluj.ro/`
Print Edition: ISSN 1584 - 286X; Online Edition: ISSN 1843 - 441X
DOI: https://doi.org/10.37193/CMI.2019.01.11

Escape from solutions stagnation. A Study on Ant System solving TSP

CAMELIA-M. PINTEA[1], BARNA IANTOVICS[2], PETRICĂ C. POP[1] and MATTHIAS DEHMER[3,4]

ABSTRACT. Nowadays, routing problems arise in different contexts of distribution of goods, transportation of commodities and people. Routing problems deals with traveling along a given network in an optimal way. One of the major goals in optimization, including optimization of routing problems, is to reduce the time of stagnation by finding an exit state. The current work is a study about the ability of ants to escape from solution stagnation on a particular routing problem, the Traveling Salesman Problem.

1. INTRODUCTION

Sometimes, during the process of solving some combinatorial optimization problems, the algorithms get stuck in local solutions. Most of the time, during search, a large number of states with the same heuristic function value are explored.

The complexity of combinatorial optimization problems (*COP*) is given among others by the complexity of the large-scale graphs and networks. In [7] a score to categorize networks based on their structural complexity is introduced. Hamson and Kibler [10] investigated when to give up searching and start again the *Boolean Satisfiability Problem*; the main result was that it was better to re-start from an initial random state than to attempt to escape from plateaus. Extensive search is used for this purpose. Restarting the algorithm for escaping from plateaus is not always a good solution. For example, regarding the *n-queens problem*, when $n > 100$, since all plateaus contain improvable states, restarting the algorithm is not necessary, see [12].

Nowadays there are few techniques to prevent blockages. Routing problems, for example the *Traveling Salesman Problem* [1, 3] and especially large-scale optimization problems as generalized versions [14], such as the *Generalized Traveling Salesman Problem* [8, 13] and the *Generalized Vehicle Routing Problem* [9], have also stagnation difficulties. In *Ant Colony Optimization (ACO)* [4], the artificial ants are agents involved in solving combinatorial optimization problems. Recently [19], there is a mathematical model that applies adaptive dynamics theory to the evolutionary dynamics of ant colonies. We will describe and test some variants of *ACO* in the case of the *Traveling Salesman Problem (TSP)* in order to see which variant provides the best strategy to escape from stagnation.

The paper is organized as follows. The next section describes the phenomenon of stagnation in combinatorial optimization problems. Different ant-based models are studied in Section *Ant systems escaping from stagnation*. The study of ant techniques for escaping from stagnation and discussions about tests results are shown in Section *Experiments and analysis of results*. The paper concludes with the benefits of the described techniques and mentions some further research.

Received: 14.04.2018. In revised form: 08.06.2018. Accepted: 15.06.2018
2010 *Mathematics Subject Classification.* 90C35, 05C85, 68T20, 68T05.
Key words and phrases. *Graphs, Traveling Salesman problem, Heuristics, Ant Colony Optimizaton.*
Corresponding author: Camelia-M. Pintea; dr.camelia.pintea@ieee.org

2. Ant systems escaping from stagnation

Metaheuristic approaches are based on two factors: "information shared among concurrently search algorithms" and "different representation level of the solution space". In order to compensate the lack of gradient in cost functions, they frequently re-start the algorithm. Sometimes fuzziness could be present [2]. One of the most efficient metaheuristic nowadays is *Ant Colony Optimization (ACO)*. In *ACO*, artificial ants build a solution to a combinatorial optimization problem by traversing a graph. The ants move from vertex to vertex along the edges of the graph, incrementally building a solution. In some variants as [3, 13] improved local update rules are used. The ants deposit an amount of pheromone on the traversed edges. The amount of deposited pheromone depends on the quality of the already found solution. The other ants use the pheromone information for guiding towards more promising edges.

Ant Colony Optimization

```
1 Set parameters, initialize pheromone trails
2 while termination conditions do not meet do
3   Construct Ant Solutions
4   Apply local search (optional)
5   Update pheromones
6 end while
7 return best solution
```

In order to study the ability of ant systems to escape from stagnation, we investigate the following variants of *ACO.*: *Ant System (AS)* [5], the first implementation of *ACO*, *MAX-MIN Ant System (MMAS)* [17], *Ant Colony System (ACS)* [3] and *Elitist Ant System (EAS)* [6].

In *Ant System (AS)*, the artificial ants have some memory, the ants are not completely blind and they will live in an environment where time is discrete. In *AS*, agents are guided by an auto-catalytic process directed by a greedy force [5].

MAX-MIN Ant System (MMAS) [17] is an improvement of *Ant System*. The most important changes are that only the best ant can update the pheromone trails, and the minimum and maximum values of the pheromone are limited.

Another improved version of *Ant System* is *Ant Colony System (ACS)* [3]. In *ACS*, a local pheromone update rule is introduced in addition to the *AS* global pheromone update rule. Dorigo et al. [6] introduced elitist ants in the *Elitist Ant System*. The elitist strategy increases the importance of the ant that found the best tour. Elitism is a daemon action by which the edges used by an ant generating the best tour from the beginning of the trial get extra pheromone. The trail of the best tour, reinforced, will influence the search of the agents.

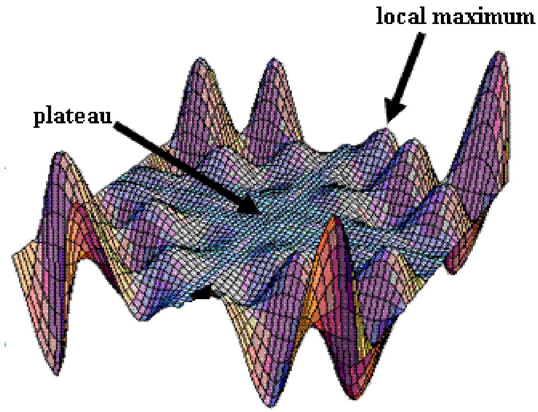
Elitism seems to better address plateaus [15] when the graph coloring problem is proposed to be solved a *Multilevel Cooperative Heuristic (MCH)*. The proposed problem is difficult to solve due to the natural expression of the cost function, with *large plateaus* in which solutions are "close' to each other having the same number of colors.

For the ant-based approach, it is important to study if elitism, when using elitist agents, can avoid stagnation. *Elitist Ant System* is going to be compared with the already mentioned ant-based systems: *Ant System*, *Ant Colony System* and *Max-Min Ant System*.

3. Experiments and analysis of results

The main contribution of the paper consists in the study of stagnation for the *Traveling Salesman Problem (TSP)*, a well known NP hard routing problem, using Ant Colony Optimization techniques. For testing *TSP*, the following TSPLIB [16] data-sets are considered:

FIGURE 1. A representation of solution space with plateaus and local and global solutions.



the 318-city problem of Lin-Kernighan, further denoted T1, and Padberg-Rinaldi data sets with 1002, denoted T2 and 2392 cities respectively, denoted T3.

Based on preliminary computational experiments, we considered the following values of the parameters: the heuristic values in the ants' solution construction, $\beta = 5$; the evaporation rate is 0.1 and there are ten ants. Tables 1-3 presents the experimental results of five consecutively runs within the maximal time of 60 seconds. A computer with AMD 2600, 1.9 GHz and 1024 MB memory was used.

The following notations are used in Tables 1-3. *Mean RPD* is the average of the runs with the relative percentage deviation ($RPD = \frac{Alg_{sol} - Min_{sol}}{Min_{sol}} \times 100$) values between the local optima versus the known optimal solution. *Max.iter.stagnation* represents the maximum number of iterations until the solution is found for each considered run. Tables 1-3 illustrate the stagnation sizes for the considered ant techniques on the given instances and also the mean value for each instance.

TABLE 1. **Stagnation study for ant-based algorithms on the Euclidean *TSP*-318-city problem of Lin-Kernighan.**

ACS		**AS**		**MMAS**		**EAS**	
Mean RPD %	Max. iter.	Mean RPD%	Max. iter.	Mean RPD %	Max. iter.	Mean RPD%	iter. iter.
2.91	81	2.99	38	2.38	42	2.53	52
2.25	80	3.22	28	2.14	52	1.57	69
4.22	71	4.01	49	2.79	30	2.24	48
1.60	82	3.70	45	2.97	68	0.88	60
1.50	94	3.69	47	1.80	35	1.95	66
2.496	*81.6*	*3.522*	*41.4*	*2.416*	*45.4*	*1.834*	*59*

Tables 1-3 illustrate stagnation sizes for the considered ant techniques on the given instances and also the mean value for each instance.

TABLE 2. **Stagnation study for ant-based algorithms on the Euclidean *TSP*-1002-city problem of Padberg-Rinaldi.**

ACS		**AS**		**MMAS**		**EAS**	
Mean RPD %	Max. iter.	Mean RPD%	Max. iter.	Mean RPD %	Max. iter.	Mean RPD%	iter. iter.
0.89	126	2.18	451	1.14	193	1.07	55
1.08	207	2.48	185	0.93	47	1.19	332
1.14	365	2.10	266	1.33	575	1.13	61
1.10	363	2.30	112	1.12	59	1.13	262
1.15	120	2.39	115	0.88	213	1.25	142

TABLE 3. **Stagnation sizes for ant-based algorithms on the Euclidean *TSP*-2392-city problem of Padberg-Rinaldi.**

ACS		**AS**		**MMAS**		**EAS**	
Mean RPD %	Max. iter.	Mean RPD%	Max. iter.	Mean RPD %	Max. iter.	Mean RPD%	iter. iter.
1.41	39	3.38	45	1.43	31	1.59	24
1.80	38	3.51	46	1.43	36	1.62	81
1.53	41	3.54	164	1.56	41	1.63	39
1.53	29	3.46	33	1.98	40	1.70	155
1.43	29	3.77	89	1.77	48	1.39	36

TABLE 4. **Statistical analysis of stagnation for routing problems for mean values in %.**

	Mean	**95% interval Mean**	**Std.Dev.**	**(Hi,Low)**	**Med.**	**AvgDMed.**
AS	3.11	2.817 to 3.412	0.652	(4.01, 2.10)	3.38	0.535
ACS	1.70	1.338 to 2.067	0.862	(4.22, 0.890)	1.50	0.509
MMAS	1.71	1.416 to 2.004	0.642	(2.97, 0.880)	1.56	0.505
EAS	1.52	1.160 to 1.889	0.455	(2.53, 0.880)	1.57	0.348

TABLE 5. **Statistical analysis of stagnation for routing problems, iterations in %.**

	Mean	**95% interval Mean**	**Std.Dev.**	**(Hi,Low)**	**Med.**	**AvgDMed.**
AS	114	59.59 to 168.8	116	(451.0, 28.0)	49.0	73.3
ACS	118	64.64 to 170.7	110	(365.0, 29.0)	81.0	68.7
MMAS	101	37.68 to 163.6	143	(575.0, 30.0)	47.0	63.5
EAS	98.8	44.19 to 153.4	89.1	(332.0, 24.0)	61.0	52.9

Tables 4 and 5 illustrate statistical analysis, for the cumulative results, including: the mean, the 95% confidence interval for mean, standard deviation, the interval (high, low) values, the median value and the average absolute deviation from median. Here are the considered simulation data, the 318-city problem of Lin-Kernighan, T1, Padberg-Rinaldi data sets with 1002, T2 and 2392 cities denoted T3.

A conclusion of the analysis is that *Elitist Ant System* has in general the smallest sizes, showing that *elitism* is in general efficient in order to escape fast from stagnation.

Figures 1 and 2 illustrate the statistical difference between the results of the algorithms, *ACS*, *AS*, *MMAS* and *EAS* related to the mean RPD and iteration values. Figure 2 illustrate the plot of the group means with 95% confidence intervals on the ACO considered algorithms for both mean and iteration values.

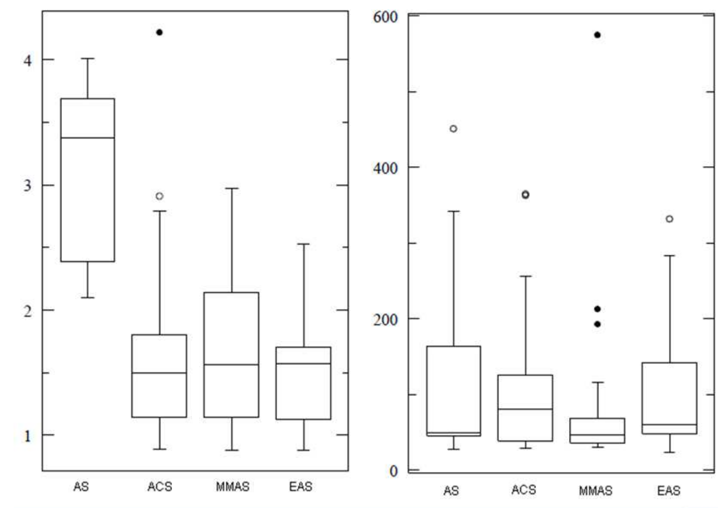


FIGURE 2. A Linear Box Plot of the considered data tested with the Ant Colony Optimization algorithms: *AS*, *ACS*, *MMAS* and *EAS*.: the mean values (left) and the iteration values (right).

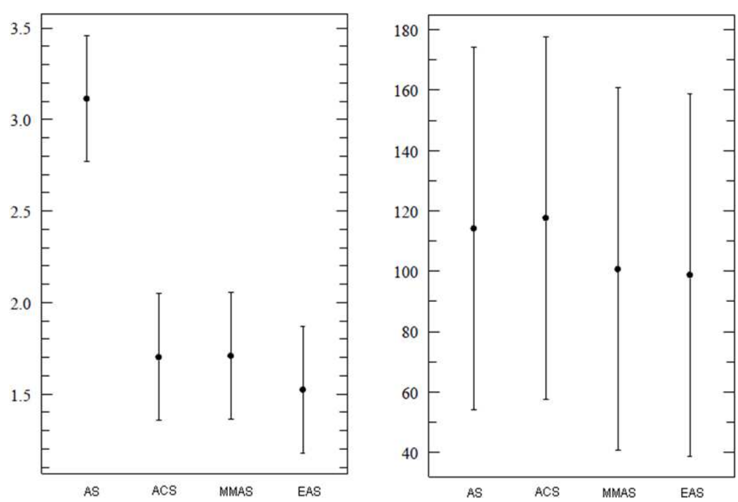


FIGURE 3. The plot of the group means with 95% confidence intervals on the considered Ant Colony Optimization algorithms: *AS*, *ACS*, *MMAS* and *EAS*; on the left side for the mean values and on the right side for the iteration values.

In particular for the 318-city problem of Lin-Kernighan, *Ant System* has achieved better results than *Elitist Ant System*, *Ant Colony System* and *Max-Min Ant System*. Here *AS* with positive feedback avoids premature convergence and also escapes relatively fast from stagnation but the final solution is not the best.

So, when analyzing the results of solving the Traveling Salesman Problem with ant algorithms, we observe that in order to escape from stagnation, under current conditions the elitism is more relevant than the local update pheromone rules.

Further research directions could include hybrid techniques with intelligent agent based system [11], ant systems, *Hill Climbing* and *Directed Plateau Search* [18]. A good strategy could be to introduce a small amount of noise to the search process or to use for example sensitive agents [13] and a combination of diversification and deterministic greedy moves.

4. Conclusions and further research

The paper includes bio-inspired techniques, ant systems, involved in a preliminary study about the ability to reduce the exploration time when solving the Traveling Salesman Problem. Several ant-based algorithms are tested and analyzed including the Elitist Ant System. The variant with elitist agents has the potential to escape fast from stagnation for the tested instances and used parameters. Further research will involve studies of hybrid techniques to escape from stagnation on a large-scale combinatorial optimization problem.

References

[1] Chvatal, V., Cook, W., Dantzig, G. B., Fulkerson, D. R. and Johnson, S. M., Solution of a Large-Scale Traveling-Salesman Problem *50 Years of Integer Programming 1958-2008* Springer, 2010, 7–28

[2] Crisan, G-C. and Nechita, E., *Solving Fuzzy TSP with Ant Algorithms*, Int. J. Comput. Commun. Control, **3** (2008), No. S, 228–231

[3] Dorigo, M. and Gambardella, L. M., *Ant Colony System: A Cooperative Learning Approach to the Traveling Salesman Problem*, IEEE Transactions on Evolutionary Computation, **1** (1997), No. 1, 53–66

[4] Dorigo, M. and Stŭtzle, T., *Ant Colony Optimization*, MIT Press, 2004

[5] Dorigo, M., Maniezzo, V. and Colorni, A., *The Ant System: Optimization by a colony of cooperating agents*, IEEE Transactions on Systems, Man, and Cybernetics, **26** (1996), Part B No. 1, 1–13

[6] Dorigo, M., Maniezzo, V. and Colorni, A., *Positive feedback as a search strategy*, Dipartimento di Elettronica, Politecnico di Milano, 1991, Technical Report, 91–016

[7] Emmert-Streib, F. and Dehmer, M., *Exploring Statistical and Population Aspects of Network Complexity*, PLoS ONE, **7** (2012), No. 5, e34523

[8] Fischetti, M., Salazar Gonzalez, J. J. and Toth, P., *The Symmetric Generalized Travelling Salesman Polytope*, Networks, **26** (1995), No. 2, 113–123

[9] Ghiani, G. and Improta, G., *An efficient transformation of the generalized vehicle routing problem*, European J. Oper. Res., **122** (2000), 11–17

[10] Hampson, S. and Kibler, D., *Plateaus and plateau search in Boolean satisfiability problems: When to give up searching and start again*, DIMACS: Discrete Math and Theoretical Computer Science, **26** (1993), 437–453

[11] Iantovics, L. B. and Enachescu, C., Intelligent Complex Evolutionary Agent-based Systems, in *Proceedings of BICS 2008 Conference, American Institute of Physics*, 2009, 116–124

[12] Minton, S., Johnston, M. D., Philips, A. B. and Laird, P., Solving large-scale constraint satisfaction and scheduling problems using a heuristic repair method, in: *Proceedings AAAI*, 1990, 17–24

[13] Pintea, C-M., *Advances in Bio-inspired Computing for Combinatorial Optimization Problem*, Springer, 2014

[14] Pop, P. C., Sitar, C. P., Zelina, I. and Taşcu, I., Exact algorithms for generalized combinatorial optimization problems. in *Combinatorial Optimization and Applications*, Springer, 2007, 54–162

[15] Prakasam, P., Toulouse, M., Crainic, T. G. and Qu, R., *Design of a Multilevel Cooperative Heuristic for the Graph Colouring Problem* Tech. Rep., 2009, CIRRELT-2009-33

[16] Reinelt, G., *TSPLIB-A Traveling Salesman Problem Library*, INFORMS Computing, **3** (1991), No. 4, 376–384

[17] Stützle, T. and Hoos, H. H., *MAX-MIN Ant System. Future Generation Computer Systems*, **16** (2000), No. 8, 889–914

[18] Sutton, A. M., Howe, A. E. and Whitley, L. D., Directed plateau search for max-k-sat, *Proceedings of the Third Annual Symposium on Combinatorial Search*, SOCS, USA, AAAI Press 2010

[19] van Oudenhove, L., Cerda, X. and Bernstein, C., An Evolutionary Dynamics Model Adapted to Eusocial Insects. PLoS ONE. 2013; 8(3): e55159. doi: 10.1371/journal.pone.0055159.

[1] Department of Mathematics and Computer Science
North University Center at Baia Mare
Technical University of Cluj-Napoca
Victoriei 76, 430122 Baia Mare, Romania
Email address: dr.camelia.pintea@ieee.org, petrica.pop@cunbm.utcluj.ro

[2] Petru Maior University
Informatics Department
Tg. Mureş, Romania
Email address: ibarna@science.upm.ro

[3] Production and Operations Management, Faculty of Management
University of Applied Sciences Upper Austria
Wehrgrabengasse 1-3, 4400 Steyr, Austria
[4] UMIT The Health and Life sciences University,
Hall in Tyrol, Austria
Email address: matthias.dehmer@umit.at