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Reverse degree distance of some graph operations

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ABSTRACT. In this paper, we present the exact formulae for the reverse degree distance of some graph operations, such as corona product, splice, link and composition of two connected graphs. Using the results obtained here, the reverse degree distance of some important classes of graphs are obtained.

1. INTRODUCTION

Let G be a simple connected graph with vertex set $V(G)$ and edge set $E(G)$. For $u \in V(G)$, let $d_G(u)$ be the degree of u in G and $d_G(u,v)$ is the distance between the vertices u and v in G. For a vertex v of G, the eccentricity $e(v)$ is the distance between v and a vertex farthest from v. The maximum eccentricity is its diameter, denoted by $d(G)$. The *degree distance* of G is defined as [14, 17, 19] $DD(G) = \frac{1}{2} \sum\limits_{u,v \in V(G)} (d_G(u) + d_G(v)) d_G(u,v)$.

It is a useful molecular descriptor [28]. Earlier as noted in [23, 27], this graph invariant appeared to be part of the molecular topological index (or Schultz index) [26], which may be expressed as $DD(G) + \sum\limits_{u \in V(G)} d_G(u)^2$, see [19, 22], where the latter part $\sum\limits_{u \in V(G)} d_G(u)^2$ is known as the first Zagreb index [20]. Thus the degree distance is also called the true Schultz index in chemical literature [12]. Tomescu [30] showed that the star is the unique graph with minimum degree distance in the class of connected graphs with n vertices. Further work on the minimum degree distance (especially for unicyclic and bicyclic graphs) may be found in A.I. Tomescu [29], Tomescu [31] and Bucicovschi and Cioaba [2]. Dankelmann et al. [11] gave asymptotically sharp upper bounds for the degree distance.

The *Wiener index* of G is, denoted by $W(G)$, defined as $W(G) = \frac{1}{2} \sum\limits_{u,v \in V(G)} d_G(u,v)$. Gutman [19] showed that if G is a tree with n vertices, then $DD(G) = 4W(G) - n(n-1)$. Thus there is no need to study the degree distance for trees because this is equivalent to the study of the Wiener index, see [13]. Balaban et al. [1] introduced the concept of *reverse Wiener index*, which is defined to be $\Lambda(G) = \frac{|V(G)|(|V(G)|-1)d(G)}{2} - W(G)$. Let $\Lambda'(G) = \frac{(|V(G)|-1)^2 d(G)}{2} - W(G)$, which is a revised version of the reverse Wiener index of G. The reverse Wiener index of unicyclic graphs are obtained by Du and Zhou [15].

The *reverse degree distance* of a connected graph G is defined in discrete mathematical chemistry as ${}^rD'(G) = 2(|V(G)|-1)\,|E(G)|\,d(G) - \frac{1}{2} \sum\limits_{u,v \in V(G)} (d_G(u) + d_G(v)) d_G(u,v)$.

Since some of the chemical graphs are derived from the graph operation for instance zig-zag nanotube is obtained from the generalized hierarchical product and 1,3- tetrameric is obtained using link of two graphs and so on. For more information refer [3, 4, 5, 6]. This

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motivative us to consider such a graph operation related to Wiener, reverse Wiener index is defined in [1] and its relation with other indices has been studied. Also the reverse degree distance and its relation with degree distance and more indices has been studied [33]. This motivate us to consider the reverse degree distance and obtained the relations between this index and Wiener index, degree distance, Zagreb index and its coindex using various graph operations such as corona product, splice, link and composition.

Some basic properties of the reverse degree distance have been established by Zhou and Trinajstic [33], and in particular, it was shown that the reverse degree distance satisfies the basic requirement to be a branching index usable in chemistry. In continuation to the study of the reverse degree distance, a natural starting point is the reverse degree distances of unicyclic graphs. In [16] the graphs with maximum reverse degrees distance in the class of unicyclic graphs with given girth, number of pendant vertices and maximum degree are determined. The degree distance and reverse degree distance of one tetragonal carbon nanocones are determined by Momen and Alaeiyan in [25].

2. Main results

The *first Zagreb index* and *second Zagreb index* are defined as $M_1(G) = \sum\limits_{u\in V(G)} d_G(u)^2$ and $M_2(G) = \sum\limits_{uv\in E(G)} d_G(u)d_G(v)$. Similarly, the *first Zagreb coindex* and *second Zagreb coindex* are defined as $\overline{M}_1(G) = \sum\limits_{uv\notin E(G)} (d_G(u) + d_G(v))$ and $\overline{M}_2(G) = \sum\limits_{uv\notin E(G)} d_G(u)d_G(v)$.

2.1. **Corona product.**

Corona product were introduced by Frucht and Harary in 1970 [18]. The *corona product* of two graphs G and H, denoted as $G \circ H$, is defined as the graph obtained by taking one copy of G and $|V(G)|$ copies of H, and joining by an edge the ith vertex of G to every vertex in the ith copy of H. Let the vertices of G and H are labeled by $v_1, v_2, \dots, v_{n_1}$ and $u_1, u_2, \dots, u_{n_2}$, respectively. For $1 \le i \le n_1$, denote by H_i the i^{th} copy of H joined to the vertex v_i and let $V(H_i) = \{u_{i1}, u_{i2}, \dots, u_{in_2}\}$. The following lemmas are follows from the structure of $G \circ H$.

Lemma 2.1. *Let G and H be two connected graphs. Then the distance between two vertices of $G \circ H$ are given as follows* $(i) d_{G\circ H}(u_{ij}, u_{pq}) = \begin{cases} 1 \text{ if } i = p \text{ and } u_j u_q \in E(H), \\ 2 \text{ if } i = p \text{ and } u_j u_q \notin E(H), \\ d_G(v_i, v_p) + 2 \text{ if } i \neq p. \end{cases}$

$(ii) d_{G\circ H}(v_i, v_p) = d_G(v_i, v_p)$, *if* $v_i, v_p \in V(G)$.

$(iii) d_{G\circ H}(v_i, u_{pq}) = d_G(v_i, v_p) + 1$, *if* $u_i \in V(G)$ *and* $u_{pq} \in V(H_i)$.

Lemma 2.2. *Let G be graph on n_1 vertices. If H is a connected graph, then the degree of a vertex of $G \circ H$ is given by* $d_{G\circ H}(x) = \begin{cases} d_G(x) + |V(H)| \text{ if } x \in V(G), \\ d_H(x) + 1 \text{ if } x \in V(H_i) \text{ for some, } i \in \{1, 2, \dots, n_1\}. \end{cases}$

Now we obtain the reverse degree distance of corona product of two connected graphs.

Theorem 2.1. *Let G and H be two connected graphs with n_1, n_2 vertices and m_1, m_2 edges, respectively. Then the reverse degree distance of $G \circ H$ is* ${}^rD'(G \circ H) = 2(n_1(n_2 + 1) - 1)(m_1 + n_1(n_2+m_2))(d(G)+2) - (n_2+1)DD(G) - 4(n_2+m_2)(n_2+1)W(G) - n_1(M_1(H)+2\overline{M}_1(H)) - \Big[2n_1(n_1 - 1)(2n_2m_2 + n_2^2) + 2n_1^2(n_2^2 + n_2 + 2m_2) + n_1(n_2^2 - n_2 - m_2 + 2n_2m_1)\Big]$.

Proof. Let $T = G \circ H$. By the definition of reverse degree distance,

$$\begin{aligned} {}^{r}D'(T) &= 2(n_1(n_2+1)-1)(m_1+n_1(n_2+m_2))(d(G)+2) \\ &\quad -\frac{1}{2}\sum_{u,v\in V(T)}(d_T(u)+d_T(v))d_T(u,v). \end{aligned} \tag{2.1}$$

We partition the set of pairs of vertices of $G \circ H$ into four subsets, namely, V_1,V_2,V_3 and V_4, where $V_1 = \{\{v_i, v_p\} \subseteq V(G\circ H) | v_i, v_p \in V(G)\}$, $V_2 = \{\{u_{ij}, u_{iq}\} \subseteq V(G\circ H) | u_{ij}, u_{iq} \in V(H_i),\ i \in \{1, 2, \ldots, n_1\}\}$, $V_3 = \{\{v_i, u_{pq}\} \subseteq V(G \circ H) | v_i \in V(G), u_{pq} \in V(H_p),\ p \in \{1, 2, \ldots, n_1\}\}$ and $V_4 = \{\{u_{ij}, u_{pq}\} \subseteq V(G \circ H) | u_{ij} \in V(H_i), u_{pq} \in V(H_p),\ i \neq p,\ i, p \in \{1, 2, \ldots, n_1\}\}$. Therefore

$$\frac{1}{2}\sum_{u,v\in V(T)}(d_T(u)+d_T(v))d_T(u,v) = S_1 + S_2 + S_3 + S_4.$$

We shall obtain the above sums are separately.

$$\begin{aligned} S_1 &= \frac{1}{2}\sum_{v_i,v_p\in V(G),\ i\neq p}(d_T(v_i)+d_T(v_p))d_T(v_i,v_p) \\ &= \frac{1}{2}\sum_{i,p=1,\ i\neq p}^{n_1}(d_G(v_i)+n_2+d_G(v_p)+n_2)d_G(v_i,v_p), \text{ by Lemmas 2.1 and 2.2} \\ &= DD(G)+2n_2W(G). \end{aligned}$$

$$\begin{aligned} S_2 &= \frac{1}{2}\sum_{i=1}^{n_1}\sum_{u_{ij},u_{iq}\in V(H_i),\ j\neq q}(d_T(u_{ij})+d_T(u_{iq}))d_T(u_{ij},u_{iq}) \\ &= \frac{1}{2}\sum_{i=1}^{n_1}\sum_{j,q=1,\ j\neq q,u_iu_q\in E(H)}^{n_2}(d_H(u_j)+d_H(u_q)+2)(1) \\ &\quad +\frac{1}{2}\sum_{i=1}^{n_1}\sum_{j,q=1,\ j\neq q,u_iu_q\notin E(H)}^{n_2}(d_H(u_j)+d_H(u_q)+2)(2), \text{ by Lemmas 2.1 and 2.2} \\ &= n_1M_1(H)+2n_1\overline{M}_1(H)+n_1n_2(n_2-1)-n_1m_2. \end{aligned}$$

$$\begin{aligned} S_3 &= \sum_{p=1}^{n_1}\sum_{i=1}^{n_1}\sum_{q=1}^{n_2}(d_T(v_i)+d_T(u_{pq}))d_T(v_i,u_{pq}) \\ &= \sum_{p=1}^{n_1}\sum_{i=1}^{n_1}\sum_{q=1}^{n_2}(d_G(v_i)+n_2+d_H(u_q)+1)(d_G(v_i,v_p)+1), \text{ by Lemmas 2.1 and 2.2} \\ &= n_2DD(G)+2n_2(n_2+1)W(G)+4m_2W(G)+2n_1n_2m_1+2n_1^2n_2(n_2+1)+4n_1^2m_2. \end{aligned}$$

$$\begin{aligned} S_4 &= \frac{1}{2}\sum_{u_{ij}\in V(H_i),u_{pq}\in V(H_p),i\neq p}(d_T(u_{ij})+d_T(u_{pq}))d_T(u_{ij},u_{pq}) \\ &= \frac{1}{2}\sum_{i,p=1,i\neq p}^{n_1}\sum_{j,q=1,j\neq q}^{n_2}(d_T(u_{ij})+d_T(u_{pq}))d_T(u_{ij},u_{pq}) \end{aligned}$$

By Lemmas 2.1 and 2.2, we have

$$\begin{aligned} S_4 &= \frac{1}{2}\sum_{i,p=1,i\neq p}^{n_1}\sum_{j,q=1,j\neq q}^{n_2}(d_H(v_j)+d_H(v_q)+2)(d_G(u_i,u_p)+2)\\ &= \frac{1}{2}\sum_{i,p=1,i\neq p}^{n_1}2(d_G(u_i,u_p)+2)\sum_{j,q=1}^{n_2}(d_H(v_j)+1)\\ &= (2n_2m_2+n_2^2)\sum_{i,p=1,i\neq p}^{n_1}(d_G(u_i,u_p)+2)\\ &= (2n_2m_2+n_2^2)(2W(G)+2n_1(n_1-1)). \end{aligned}$$

Using the sums S_1 to S_4 in (2.1), we obtain:

$$\begin{aligned} {}^rD'(T) &= 2(n_1(n_2+1)-1)(m_1+n_1(n_2+m_2))(d(G)+2)-(n_2+1)DD(G)\\ &\quad -4(n_2+m_2)(n_2+1)W(G)-n_1(M_1(H)+2\overline{M}_1(H))-\Big[2n_1(n_1-1)(2n_2m_2\\ &\quad +n_2^2)+2n_1^2(n_2^2+n_2+2m_2)+n_1(n_2^2-n_2-m_2+2n_2m_1)\Big]. \end{aligned}$$

□

Let C_n,P_n and K_n denote the cycle, path and complete graph on n vertices, respectively. It is known that $W(P_n)=\frac{n(n^2-1)}{6}$, $PI(C_n)=\frac{n^3}{8}$ when n is even, and $\frac{n(n^2-1)}{8}$ otherwise and $W(K_n)=\frac{n(n-1)}{2}$. It can be easily verified that $DD(P_n)=\frac{n(n-1)(2n-1)}{3}$,$DD(C_n)=\frac{n^3}{2}$ when n is even, and $\frac{n(n^2-1)}{2}$ otherwise and $DD(K_n)=n(n-1)^2$.

By direct calculations we obtain the first and second Zagreb indices of P_n and C_n. $M_1(C_n)=4n,\ n\geq 3$, $M_1(P_1)=0$, $M_1(P_n)=4n-6,\ n>1$ and $M_1(K_n)=n(n-1)^2$. $M_2(P_n)=4(n-2)$ and $M_2(C_n)=4n$. Similarly, $\overline{M}_1(P_n)=2(n^2-4n+4)$, $\overline{M}_1(C_n)=2n(n-3)$ and $\overline{M}_1(K_n)=0$.

For a given graph G, its $t-$*fold bristled* graph $Brs_t(G)$ is obtained by attaching t vertices of degree 1 to each vertex of G. This graph can be represented as the corona product of G and complement of a complete graph on t vertices.

Example 2.1. Let G be a graph with n vertices. Then ${}^rD'(G\circ\overline{K}_t)=2(nt+n-1)(m+nt)(d(G)+2)-(t+1)DD(G)-4t(t+1)W(G)-2nmt-nt(3nt+n-2)$.

Example 2.2. (i) ${}^rD'(P_n\circ\overline{K}_t)=2(nt+n-1)^2(n+1)-\frac{n}{3}(n-1)(t+1)(2nt+2n+2t-1)+nt(3nt+3n-4)$.

$$(ii)\,{}^rD'(C_n\circ\overline{K}_t)=\begin{cases} n(n+4)(t+1)(nt+n-1)-\frac{n^3(t+1)^2}{2}\\ -2n^2t-nt(3nt+n-2)\ \text{ if } n \text{ is even}\\ n(n+3)(t+1)(nt+n-1)-\frac{n(n^2-1)(t+1)^2}{2}\\ -2n^2t-nt(3nt+n-2)\ \text{ if } n \text{ is odd.}\end{cases}$$

(iii) A special corona graph $C_n\circ K_1$ is called a *sunlet* graph on $2n$ vertices. ${}^rD'(C_n\circ K_1)=2n(n^2+4n-3)$ if n is even and $2n(3n^2+2n-1)$ if n is odd.

The *star* graph S_{t+1} on $t+1$ vertices is the corona product of K_1 and $\overline{K}_t$. The *fan* graph F_{t+1} and the *wheel* graph W_{t+1} on $t+1$ vertices are also corona product of K_1 and P_t and C_t. From the above formula the reverse degree distance of these graphs are obtained.

Example 2.3. $(i)^rD'(K_1\circ\overline{K}_t)=t(7t-1)$. $(ii)^rD'(K_1\circ P_t)=3t^2-22t+25$. $(iii)^rD'(K_1\circ C_t)=3t^2-20t$.

Example 2.4. Using Theorem 2.1, we have $(i)\ {}^rD'(P_n \circ \overline{K}_2) = 12n^3 + 27n^2 - 15n + 2$.
$(ii){}^rD'(P_n \circ K_2) = 2n^2t - 6n^2t + 2n^2t^3 + 5nt + 2nt^3 + 3nt^2 + 2n^3 - 6n^2 - 3n - 6t^2 - 12t - 4$.
$(iii){}^rD'(P_n \circ C_m) = 4n^3t^2 + \frac{16n^3t}{3} + \frac{4n^3}{3} + 7n^2 - \frac{7n}{3} - 4n^2t^2 + \frac{17nt}{3} - 7n^2t + nt^2 + 2$.

2.2. **Splice and link.** Let G and H be two simple connected graphs with the vertex sets $V(G)$ and $V(H)$ and the edge sets $E(G)$ and $E(H)$, respectively. For given vertices $y \in V(G)$ and $z \in V(H)$, a *splice* of G and H by vertices y and z is denoted by $S(G.H)(y,z)$ and defined by identifying the vertices y and z in the union of G and H. In the following Lemma, the distance between vertices of $S(G.\,H)(y,z)$ is computed. The proof can be easily obtained from the definition of splice of graphs, so is omitted.

Lemma 2.3. *Let G and H be two graphs. Then*

$$d_{S(G.\,H)(y,z)}(u,v) = \begin{cases} d_G(u,v)\ if\ u,v \in V(G), \\ d_H(u,v)\ if\ u,v \in V(H), \\ d_G(u,y) + d_H(z,v)\ if\ u \in V(G)\ v \in V(H). \end{cases}$$

Theorem 2.2. *Let G and H be two graphs with n_1, n_2 vertices and m_1, m_2 edges. Then*
$${}^rD'(S(G.\,H)(y,z)) = 2(n_1+n_2-1)(m_1+m_2)(d(y)+d(z)) - DD(G) - DD(H)$$
$$-(n_2-1)\sum_{y\neq u\in V(G)} d_G(u)d_G(u,y) - (n_1-1)\sum_{z\neq v\in V(H)} d_H(v)d_H(v,z) - 2m_1D(z)$$
$-2m_2D(y)$, *where* $D(y) = \sum_{x\in V(G)} d_G(x,y)$ *and* $D(z) = \sum_{z\in V(H)} d_H(x,z)$.

Proof. Let $S = S(G.\,H)(y,z)$. By the definition of reverse degree distance,

$${}^rD'(S) = 2(n_1+n_2-1)(m_1+m_2)(d(y)+d(z)) - \frac{1}{2}\sum_{u,v\in V(S)} (d_S(u)+d_S(v))d_S(u,v). \quad (2.2)$$

We partition the set of pairs of vertices of S into five subsets, namely, V_1,V_2,V_3,V_4, and V_5, where $V_1 = \{\{u,v\} \subseteq V(S) | u,v \neq y \in V(G)\}$, $V_2 = \{\{u,v\} \subseteq V(S) | u = y, v \in V(G)\}$, $V_3 = \{\{u,v\} \subseteq V(S) | u,v \neq z \in V(H)\}$, $V_4 = \{\{u,v\} \subseteq V(S) | u = z, v \in V(H)\}$ and $V_5 = \{\{u,v\} \subseteq V(S) | y \neq u \in V(G), z \neq v \in V(H)\}$.

Therefore by Lemma 2.3, we have

$$\begin{aligned}
&\frac{1}{2}\sum_{u,v\in V(S)} (d_S(u)+d_S(v))d_S(u,v) = \frac{1}{2}\sum_{u,v}(d_S(u)+d_S(v))d_S(u,v) \\
= &\frac{1}{2}\sum_{u,v\neq y,\ u,v\in V(G)} (d_G(u)+d_G(v))d_G(u,v) + \frac{1}{2}\sum_{u=y,\ v\in V(G)} (d_G(y)+d_H(z)+d_G(v))d_G(v,y) \\
+ &\frac{1}{2}\sum_{u,v\neq z,\ u,v\in V(H)} (d_H(u)+d_H(v))d_H(u,v) + \frac{1}{2}\sum_{u=z,\ v\in V(H)} (d_G(y)+d_H(z)+d_H(v))d_H(v,z) \\
+ &\frac{1}{2}\sum_{y\neq u\in V(G),\ z\neq v\in V(H)} (d_G(u)+d_H(v))(d_G(u,y)+d_H(v,z)) \\
= &DD(G)+DD(H)+(n_2-1)\sum_{y\neq u\in V(G)} d_G(u)d_G(u,y) \\
&+(n_1-1)\sum_{z\neq v\in V(H)} d_H(v)d_H(v,z) + 2m_1D(z) + 2m_2D(y). \quad (2.3)
\end{aligned}$$

Using (2.3) in (2.2), we obtain:

$$\begin{aligned}
{}^rD'(S) = &\ 2(n_1+n_2-1)(m_1+m_2)(d(y)+d(z)) - DD(G) - DD(H) \\
&-(n_2-1)\sum_{y\neq u\in V(G)} d_G(u)d_G(u,y) - (n_1-1)\sum_{z\neq v\in V(H)} d_H(v)d_H(v,z) \\
&-2m_1D(z) - 2m_2D(y).
\end{aligned}$$

□

Let G and H be two simple connected graphs with the vertex sets $V(G)$ and $V(H)$ and the edge sets $E(G)$ and $E(H)$, respectively. For vertices $y \in V(G)$ and $z \in V(H)$, a *link* of G and H by vertices y and z is denoted by $L(G \bullet H)(y,z)$ and obtained by joining y and z by an edge in the union of these grap. In the following Lemma, the distance between vertices of $L(G \bullet H)(y,z)$ is computed. The proof can be easily obtained from the definition of link of graphs, so is omitted.

Lemma 2.4. *Let G and H be two graphs. Then*

$$d_{L(G\bullet H)(y,z)}(u,v) = \begin{cases} d_G(u,v)\ if\ u,v \in V(G), \\ d_H(u,v)\ if\ u,v \in V(H), \\ d_G(u,y) + d_H(z,v) + 1\ if\ u \in V(G)\ v \in V(H). \end{cases}$$

Theorem 2.3. *Let G and H be two graphs with n_1, n_2 vertices and m_1, m_2 edges. Then* ${}^rD'(L(G\bullet H)(y,z)) = 2(n_1+n_2)(m_1+m_2+1)(d(y)+d(z)+1) - DD(G) - DD(H) - n_2 \sum\limits_{y\neq u \in V(G)} d_G(u)d_G(u,y) - n_1 \sum\limits_{z\neq v\in V(H)} d_H(v)d_H(v,z) - \Big[(2m_1+2)D(z) + (2m_2+2)D(y)\Big] - \Big[(n_2-1)(2m_1+1) + (n_1-1)(2m_2+1) + 2m_1 + 2m_2 + 2\Big]$, *where* $D(y) = \sum\limits_{x\in V(G)} d_G(x,y)$ *and* $D(z) = \sum\limits_{z\in V(H)} d_H(x,z)$.

Proof. Let $L = L(G \bullet H)(y,z)$. By the definition of reverse degree distance,

$$ {}^rD'(L) = 2(n_1+n_2)(m_1+m_2+1)(d(y)+d(z)+1) - \frac{1}{2}\sum_{u,v\in V(L)} (d_L(u)+d_L(v))d_L(u,v). \quad (2.4)$$

By Lemma 2.4, we have

$$\begin{aligned}
& \frac{1}{2}\sum_{u,v}(d_S(u)+d_S(v))d_S(u,v) \\
= \ & \frac{1}{2}\sum_{u,v\neq y,\ u,v\in V(G)} (d_G(u)+d_G(v))d_G(u,v) + \frac{1}{2}\sum_{u=y,\ v\in V(G)} (d_G(y)+1+d_G(v))d_G(v,y) \\
& +\frac{1}{2}\sum_{u,v\neq z,\ u,v\in V(H)} (d_H(u)+d_H(v))d_H(u,v) + \frac{1}{2}\sum_{u=z,\ v\in V(H)} (1+d_H(z)+d_H(v))d_H(v,z) \\
& + \sum_{v=z, u\neq y\in V(G)} (1+d_H(z)+d_G(u))(1+d_G(u,y)) \\
& +\frac{1}{2}\sum_{y\neq u\in V(G),\ z\neq v\in V(H)} \Big((d_G(u)+d_H(v))(d_G(u,y)+d_H(v,z)+1)\Big) + d_G(y) + d_H(z) + 2 \\
= \ & n_2 \sum_{y\neq u\in V(G)} d_G(u)d_G(u,y) + n_1 \sum_{z\neq v\in V(H)} d_H(v)d_H(v,z) + (2m_1+2)D(z) \\
& +DD(G) + (2m_2+2)D(y) + DD(H) + (n_2-1)(2m_1+1) + 2m_1 \\
& +(n_1-1)(2m_2+1) + 2m_2 + 2. \qquad (2.5)
\end{aligned}$$

Using (2.5) in (2.4), we have

$$\begin{aligned}
{}^rD'(L) &= 2(n_1+n_2)(m_1+m_2+1)(d(y)+d(z)+1) - DD(G) - DD(H) \\
& -n_2 \sum_{y\neq u\in V(G)} d_G(u)d_G(u,y) - n_1 \sum_{z\neq v\in V(H)} d_H(v)d_H(v,z) - \Big[(2m_1+2)D(z) \\
& +(2m_2+2)D(y)\Big] - \Big[(n_2-1)(2m_1+1) + (n_1-1)(2m_2+1) + 2m_1 + 2m_2 + 2\Big].
\end{aligned}$$

□

2.3. **Composition.** The *composition* of the graphs G and H, denoted by $G[H]$, has vertex set $V(G) \times V(H)$ in which $(u_1, v_1)(u_2, v_2)$ is an edge whenever u_1u_2 is an edge in G or, $u_1 = u_2$ and v_1v_2 is an edge in H. The following lemma gives the distance and degree of vertices of $G[H]$.

Lemma 2.5. *Let G and H be two connected graphs with $V(G) = \{u_1, u_2, \ldots, u_{n_1}\}$ and $V(H) = \{v_1, v_2, \ldots, v_{n_2}\}$. If x_{ij} denote the vertex (u_i, v_j) of $G[H]$, then (i) the distance between two vertices of $G[H]$ is given by* $d_{G[H]}(x_{ij}, x_{pq}) = \begin{cases} d_G(u_i, u_p) \ if \ j = q \text{ or } i \neq p \\ 1 \ if \ i = p \text{ and } v_jv_q \in E(H) \\ 2 \ if \ i = p \text{ and } v_jv_q \notin E(H). \end{cases}$
(ii) the degree of a vertex x_{ij} of $G[H]$ is $n_2d_G(u_i) + d_H(v_j)$.

Theorem 2.4. *Let G and H be two connected graphs with n_1 and n_2 vertices, respectively. Then ${}^rD'(G[H]) = 2(n_1n_2 - 1)(m_1n_2^2 + m_2n_1)d(G) - n_2^3DD(G) - 2(2m_2 + M_1(H) + \overline{M}_1(H))W(G) - n_1M_1(H) - 2n_1\overline{M}_1(H) - 4n_2m_1(n_2^2 - n_2 - m_2)$.*

Proof. Let $V(G) = \{u_1, u_2, \ldots, u_{n_1}\}$, similarly for $V(H) = \{v_1, v_2, \ldots, v_{n_2}\}$. Let x_{ij} denote the vertex (u_i, v_j) of $G[H]$. By the definition of reverse degree distance,

$$\begin{aligned} {}^rD'(G[H]) &= 2(n_1n_2 - 1)(m_1n_2^2 + m_2n_1)d(G) \\ &\quad -\frac{1}{2}\sum_{i=1}^{n_1-1}\sum_{\substack{j,\ell=1\\ j\neq \ell}}^{n_2}(d_{G[H]}(x_{ij}) + d_{G[H]}(x_{i\ell}))d_{G[H]}(x_{ij}, x_{i\ell}) \\ &\quad -\frac{1}{2}\sum_{\substack{i,k=1\\ i\neq k}}^{n_1}\sum_{j=1}^{n_2}(d_{G[H]}(x_{ij}) + d_{G[H]}(x_{kj}))d_{G[H]}(x_{ij}, x_{kj}) \\ &\quad -\frac{1}{2}\sum_{\substack{i,k=1\\ i\neq k}}^{n_1}\sum_{\substack{j,\ell=1\\ j\neq \ell}}^{n_2}(d_{G[H]}(x_{ij}) + d_{G[H]}(x_{k\ell}))d_{G[H]}(x_{ij}, x_{k\ell}). \end{aligned} \tag{2.6}$$

We partition the sums into three sums, S_1,S_2 and S_3 as follows.

$$\begin{aligned} S_1 &= \sum_{i=1}^{n_1}\sum_{\substack{j,\ell=1\\ j\neq \ell}}^{n_2}(d_{G[H]}(x_{ij}) + d_{G[H]}(x_{i\ell}))d_{G[H]}(x_{ij}, x_{i\ell}) \\ &= \sum_{i=1}^{n_1}\sum_{\substack{j,\ell=1\\ j\neq \ell}}^{n_2}(2n_2d_G(u_i) + d_H(v_j) + d_H(v_\ell))d_{G[H]}(x_{ij}, x_{i\ell}) \\ &= 2n_2\sum_{i=1}^{n_1}d_G(u_i)\Big(\sum_{v_jv_\ell\in E(H)}d_H(v_j, v_\ell) + \sum_{v_jv_\ell\notin E(H)}d_H(v_j, v_\ell)\Big) \\ &\quad +\sum_{i=1}^{n_1}\Big(\sum_{v_jv_\ell\in E(H)}(d_H(v_j) + d_H(v_\ell)d_H(v_j, v_\ell) + \sum_{v_jv_\ell\notin E(H)}(d_H(v_j) + d_H(v_\ell)d_H(v_j, v_\ell)\Big), \end{aligned}$$

since $d_{G[H]}(x_{ij}, x_{i\ell}) = 1$ if $v_jv_\ell \in E(H)$ and 2 if $v_jv_\ell \notin E(H)$.

$$
\begin{aligned}
&= 4n_2m_1\left(\sum_{v_j\in V(H)} d_H(v_j) + 2\sum_{v_j\in V(H)}(n_2 - d_H(v_j) - 1)\right)\\
&\quad + \sum_{i=1}^{n_1}\left(\sum_{v_jv_\ell\in E(H)}(d_H(v_j)+d_H(v_\ell)) + \sum_{v_jv_\ell\notin E(H)}(d_H(v_j)+d_H(v_\ell))\right),\\
&= 4n_2m_1\Big(2m_2 + 2(n_2^2 - 2m_2 - n_2)\Big) + 2n_1M_1(H) + 4n_1\overline{M}_1(H).
\end{aligned}
$$

$$
\begin{aligned}
S_2 &= \sum_{\substack{i,k=1\\ i\neq k}}^{n_1}\sum_{j=1}^{n_2}(d(x_{ij}) + d(x_{kj}))d_{G[H]}(x_{ij}, x_{kj})\\
&= \sum_{\substack{i,k=1\\ i\neq k}}^{n_1}\sum_{j=1}^{n_2}(n_2(d(u_i)+d(u_k)) + 2d(v_j))d_G(u_i,u_k)\\
&= \sum_{\substack{i,k=1\\ i\neq k}}^{n_1}\sum_{j=1}^{n_2} n_2(d(u_i)+d(u_k))d_G(u_i,u_k) + \sum_{\substack{i,k=1\\ i\neq k}}^{n_1}\sum_{j=1}^{n_2} 2d(v_j)d_G(u_i,u_k)\\
&= 2n_2^2DD(G) + 8m_2W(G).
\end{aligned}
$$

$$
\begin{aligned}
S_3 &= \sum_{\substack{i,k=1\\ i\neq k}}^{n_1}\sum_{\substack{j,\ell=1\\ j\neq \ell}}^{n_2}(d(x_{ij}) + d(x_{k\ell}))d_{G[H]}(x_{ij}, x_{k\ell})\\
&= \sum_{\substack{i,k=1\\ i\neq k}}^{n_1}\sum_{\substack{j,\ell=1\\ j\neq \ell}}^{n_2}(n_2d(u_i) + d(v_j) + n_2d(u_k) + d(v_\ell))d_G(u_i,u_k),\\
&\quad \text{since } d_{G[H]}(x_{ij}, x_{k\ell}) = d_G(u_i,u_k) \text{ for all } j \text{ and } k \text{ and further the distance}\\
&\quad \text{between the corresponding vertices of the layers is counted in } S_2\\
&= 2n_2^2(n_2-1)DD(G) + 4W(G)(M_1(H) + \overline{M}_1(H)).
\end{aligned}
$$

Using S_1 to S_3 in (2.6), we have

$$
\begin{aligned}
{}^rD'(G[H]) &= 2(n_1n_2-1)(m_1n_2^2 + m_2n_1)d(G) - n_2^3DD(G) - 2(2m_2 + M_1(H)\\
&\quad + \overline{M}_1(H))W(G) - n_1M_1(H) - 2n_1\overline{M}_1(H) - 4n_2m_1(n_2^2 - n_2 - m_2).
\end{aligned}
$$

□

As an application we present formulae for reverse degree distance of open and closed fences, $P_n[K_2]$ and $C_n[K_2]$.

Example 2.5.

$(i)^rD'(P_n[K_2]) = \frac{2}{3}(6n^2 - 9n - 1) - \frac{4}{3}n(n-1)(n+7)$.

$$(ii)^rD'(C_n[K_2]) = \begin{cases} n(5n^2-5n-4) \ \textit{if } n \textit{ is even}\\ 5n^2(n^2-3) \ \textit{if } n \textit{ is odd.}\end{cases}$$

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