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Some new types of decomposition of continuity II

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ABSTRACT. In this paper we extended (or generalized) the notions studied in [Carpintero, C. Moreno, J. and Rosas, E., *Some New type of decomposition of continuity*, submitted]. Using the notion of μ-regular open set and μ-semi regular open set, we introduce the concept of locally μ-regular semi closed sets and locally μ-semi regular semi closed sets and give a new theory of decomposition of continuity and some weak forms of continuity are studied. Also we improve some recent results due by [Roy, B. and Sen, R., *On decomposition of weak continuity*, Creat. Math. Inform, **24** (2015), No. 1-2, 83–88].

1. INTRODUCTION AND PRELIMINARIES

In 2002, A. Császar [2], introduced the notions of generalized topology and generalized continuity. It is observed that a large numbers of articles are devoted to the study of generalized open sets and certain type of sets associated to a topological spaces, containing the class of open sets and possessing properties more or less to those open sets. We recall some notions defined in [2]. Let X be a nonempty set and let exp$X$, denote the power set of X. We call a class $\mu \subseteq$ exp$X$ a generalized topology [2] (briefly, GT) if $\emptyset \in \mu$ and the union of elements of μ belong to μ. A set X with a GT μ on it is called a generalized topological space(briefly, GTS) and is denoted by (X, μ). The elements of μ are called μ-open sets and the complement of a μ-open sets are called μ-closed sets. For $A \subset X$, we denote by $c_\mu(A)$ the intersection of all μ-closed sets containing A, i.e., the smallest μ-closed set containing A; and $i_\mu(A)$, the union of all μ-open sets contained in A, i.e., the largest μ-open set contained in A (see [2],[3]). It is easy to observe that i_μ and c_μ are idempotent and monotone (ρ : exp$X \mapsto$ exp$X$ is said idempotent if $A \subset X$ implies $\rho(\rho(A)) = \rho(A)$ and monotone, if $A \subset B$ implies $\rho(A) \subseteq \rho(B)$) [4]. It is also well known (see [2],[3]), that if μ is a GT on X, $x \in X$ and $A \subset X$, then $x \in c_\mu(A)$ if and only if $x \in M$ and $M \in \mu$, implies $M \cap A \neq \emptyset$ and $c_\mu(X \backslash A) = X \backslash i_\mu(A)$. In the same form as in topological space, are defined the notion of semi open sets, we obtain that if (X, μ) is a generalized topological space, $A \subseteq X$ is said to be μ-semi open if there exists a μ-open set W such that $W \subseteq A \subseteq c_\mu(W)$ or equivalently, A is μ-semi open if and only if $A \subseteq c_\mu(i_\mu(A))$, the collection of all μ-semi open set in X is denoted by μ-$SO(X)$. The complement of a μ-semi open set is called μ-semi closed, the collection of all μ-semi closed set in X is denoted by μ-$SC(X)$. For $A \subset X$, we denote by $sc_\mu(A)$ the intersection of all μ-semi closed sets containing A, i.e., the smallest μ-semi closed set containing A; and $si_\mu(A)$, the union of all μ-semi open sets contained in A, i.e., the largest μ-semi open set contained in A (see [7]). sc_μ is idempotent and monotone. Also $sc_\mu(A) = A \cup i_\mu(c_\mu(A))$, $sc_\mu(X - A) = X - si_\mu(A)$ and $si_\mu(X - A) = X - sc_\mu(A)$ [7]. Throughout the paper (X, τ) and (Y, β) will represent topological spaces and μ is a GT on a topological space (X, τ). A subset A of a topological space (X, τ) is called semi regular open if there exists a regular open set V that is ($V = int(cl(V))$) such that $V \subseteq A \subseteq cl(V)$. The collection of all regular open sets ia a topological space is denoted by $RO(X)$ and

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the collection of all semi regular open sets is a topological space is denoted by $SRO(X)$. In this article, using the notion of μ-regular open sets and μ-regular semi open sets, we introduce the concept of locally μ-regular semi closed sets as a generalization of locally μ-regular closed set introduced in [6]. Also we introduce the notions of locally μ-semi regular semi closed sets and give a new theory of decomposition of continuity and some weak forms are studied.

2. Locally μ-semi regular semi closed sets

Definition 2.1. [6] Let μ be a GT on a topological space (X, τ). A subset A of X is called locally μ-regular closed if $A = U \cap F$ where $U \in RO(X)$ and F is μ-closed.

Remark 2.1. If μ is a GT on a topological space (X, τ), then

1. Every regular open set as well as a μ-closed set is locally μ-regular closed.
2. A is locally μ-regular closed if and only if $X - A$ is the union of a regular closed and a μ-open set.
3. Finite intersection of locally μ-regular closed is locally μ-regular closed.

Definition 2.2. [5] Let μ be a GT on a topological space (X, τ). A subset A of X is called locally μ-closed if $A = U \cap F$ where $U \in \tau$ and F is μ-closed.

Remark 2.2. Every locally μ-regular closed is locally μ-closed, but the converse is not necessarily true.

Definition 2.3. Let μ be a GT on a topological space (X, τ). A subset A of X is called locally μ-regular semi closed (briefly locally μ-rsclosed) if $A = U \cap F$ where $U \in RO(X)$ and F is μ-semi closed.

Remark 2.3. If μ is a GT on a topological space (X, τ), then

1. Every regular open set as well as a μ-closed set is locally μ-regular semi closed.
2. A is locally μ-regular semi closed if and only if $X - A$ is the union of a regular closed and a μ-semi open set.
3. Finite intersection of locally μ-regular semi closed is locally μ-regular semi closed.

Definition 2.4. [1] Let μ be a GT on a topological space (X, τ). A subset A of X is called locally μ-semi closed (briefly locally μ-sclosed) if $A = U \cap F$ where $U \in \tau$ and F is μ-semi closed.

Remark 2.4. If μ is a GT on a topological space (X, τ), then

1. Every locally μ-regular semi closed is locally μ-semi closed, but the converse is not necessarily true.
2. Every locally μ-regular closed is locally μ-regular semi closed, but the converse is not necessarily true.

Example 2.1. Let $X = \{a, b, c\}$ with topology $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$ and GT $\mu = \{\emptyset, \{b, c\}\}$.

1. The set of locally μ-semi closed sets =$\{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}\}$.
2. The set of locally μ regular semi closed = $\{\emptyset, X, \{a\}, \{b\}, \{b, c\}\}$.

Theorem 2.1. *Let μ be a GT on a topological space (X, τ). $A \subseteq X$ is locally μ-regular semi closed if and only if there exists a regular open set U such that $A = U \cap sc_\mu(A)$.*

Proof. Let A be a locally μ-regular semi closed subset of X, then $A = U \cap F$, where $U \in RO(X)$ and F is μ-semi closed. Follows that $A = A \cap U \subseteq U \cap sc_\mu(A) \subseteq U \cap sc_\mu(F) = U \cap F = A$. In consequence, $A = U \cap sc_\mu(A)$. Conversely. Since $sc_\mu(A)$ is a μ-semi closed. Follows that A is locally μ-semi closed. □

Theorem 2.2. *Let μ be a GT on a topological space (X, τ). If $A \subseteq B \subseteq X$ and B is locally μ-regular semi closed, then there exists a locally μ-regular semi closed C such that $A \subseteq C \subseteq B$.*

Proof. Suppose that B is locally μ-regular semi closed, by Theorem 2.1, $B = U \cap sc_\mu(B)$, where U is a regular open. Follows $A \subseteq B \subseteq U$, in consequence, $A \subseteq U \cap sc_\mu(B)$, thus $A \subseteq U \cap sc_\mu(A)$. If we take $C = U \cap sc_\mu(A)$, C is locally μ-regular semi closed and $A \subseteq C \subseteq B$. □

There exist some relation between the generalized topology μ and the topology of the space in order to decide if A is locally μ-regular semi closed, under what conditions $sc_\mu(A) - A$ is locally μ-regular semi closed. Recall that $sc_\mu(A) = A \cup i_\mu(c_\mu(A))$. In the case that $RO(X) \subset \mu$, we have the following theorem.

Theorem 2.3. *Let μ be a GT on a topological space (X, τ) such that $RO(X) \subset \mu$. If A is locally μ-regular semi closed. Then:*

1. *$sc_\mu(A) - A$ is μ-semi closed.*
2. *$A \cup (X - sc_\mu(A))$ is μ-semi open set.*
3. *A is contained in $si_\mu(A \cup (X - sc_\mu(A)))$.*

Proof. 1.-Suppose that A is locally μ-regular semi closed subset of X, then there exists a regular open set U such that $A = U \cap sc_\mu(A)$. Follows that: $sc_\mu(A) - A = sc_\mu(A) - (U \cap sc_\mu(A)) = sc_\mu(A) \cap (X - (U \cap sc_\mu(A))) = sc_\mu(A) \cap ((X - U) \cup (X - sc_\mu(A))) = sc_\mu(A) \cup (X - U) \cap sc_\mu(A) \cap (X - sc_\mu(A)) = sc_\mu(A) \cap (X - U)$. Now $sc_\mu(A)$ is μ-semi closed and $X - U$ is regular closed and $RO(X) \subset \mu$, we obtain that $X - U$ and then $sc_\mu(A) \cap (X - U)$ is μ-semi closed.
2.- Using (1), $sc_\mu(A) - A$ is μ-semi closed, then its complement $X - (sc_\mu(A) - A)$ is μ-semi open, but $X - (sc_\mu(A) - A) = X - (sc_\mu(A) \cap (X - A) = A \cup (X - sc_\mu(A))$.
3.-Using (2), $A \subset (A \cup (X - sc_\mu(A))) = si_\mu(A \cup (X - sc_\mu(A)))$. □

Remark 2.5. The next example shows that $RO(X) \subset \mu$ is necessary in the above theorem.

Example 2.2. Let $X = \{a, b, c\}$ with topology $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$ and GT $\mu = \{\emptyset, \{b, c\}\}$. Observe that $RO(X) = \{\emptyset, X, \{a\}, \{b\}\}$. If we take $A = \{c\}$, $sc_\mu(A) = \{b, c\}$ and $sc_\mu(A) - A = \{b\}$ is not μ-semi closed.

Definition 2.5. Let μ be a GT on a topological space (X, τ). A subset A of X is called locally μ-semi regular semi closed (briefly locally μ-srsclosed) if $A = U \cap F$ where $U \in SRO(X)$ and F is μ-semi closed.

Remark 2.6. If μ is a GT on a topological space (X, τ), then

1. Every semi regular open set as well as a μ-semi closed set is locally μ-semi regular semi closed.
2. A is locally μ-semi regular semi closed if and only if $X - A$ is the union of a semi regular closed and a μ-semi open set.
3. Finite intersection of locally μ-semi regular semi closed is locally μ-semi regular semi closed.

Remark 2.7. Every locally μ-regular semi closed is locally μ-semi regular semi closed, but the converse is not necessarily true.

Example 2.3. Let $X = \{a, b, c\}$ with topology $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$ and GT $\mu = \{\emptyset, \{b, c\}\}$.

1. Locally μ regular semi closed = $\{\emptyset, X, \{a\}, \{b\}, \{b, c\}\}$.
2. Locally μ semi regular semi closed =$\{\emptyset, X, \{a\}, \{b\}, \{c\}, \{a, c\}, \{b, c\}\}$.

Theorem 2.4. *Let μ be a GT on a topological space (X, τ). $A \subseteq X$ is locally μ-semi regular semi closed if and only if there exists a semi regular open set U such that $A = U \cap sc_\mu(A)$.*

Proof. Let A be a locally μ-semi regular semi closed subset of X, then $A = U \cap F$, where $U \in SRO(X)$ and F is μ-semi closed. Follows that $A = A \cap U \subseteq U \cap sc_\mu(A) \subseteq U \cap sc_\mu(F) = U \cap F = A$. In consequence, $A = U \cap sc_\mu(A)$. Conversely, since $sc_\mu(A)$ is a μ-semi closed. Follows that A is locally μ-semi regular semi closed. □

Remark 2.8. Every locally μ-regular semi closed is locally μ-semi closed but the converse is false as shown by the next example.

Example 2.4. Let $X = \{a, b, c\}$ with topology $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$ and GT $\mu = \{\emptyset, \{b, c\}\}$.

1. The set of locally μ-semi closed sets $=\{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}\}$.
2. The set of locally μ regular semi closed = $\{\emptyset, X, \{a\}, \{b\}, \{b, c\}\}$.

Theorem 2.5. *Let μ be a GT on a topological space (X, τ). If $A \subseteq B \subseteq X$ and B is locally μ-semi regular semi closed, then there exists a locally μ-semi regular semi closed C such that $A \subseteq C \subseteq B$.*

Proof. Suppose that B is locally μ-semi regular semi closed, by Theorem 2.4, $B = U \cap sc_\mu(B)$, where U is a semi regular open. Follows $A \subseteq B \subseteq U$, in consequence, $A \subseteq U \cap sc_\mu(B)$, Thus $A \subseteq U \cap sc_\mu(A)$. If we take $C = U \cap sc_\mu(A)$, C is locally μ-semi regular semi closed and $A \subseteq C \subseteq B$. □

There exist some relation between the generalized topology μ and the topology of the space in order to decide if A is locally μ-semi regular semi closed, under what conditions $sc_\mu(A) - A$ is locally μ-semi regular semi closed. In the case that $SRO(X) \subset \mu$, we have the following theorem.

Theorem 2.6. *Let μ be a GT on a topological space (X, τ). If A is locally μ-semi regular semi closed such that $SRO(X) \subset \mu$. Then:*

1. *$sc_\mu(A) - A$ is μ-semi closed.*
2. *$A \cup (X - sc_\mu(A))$ is μ-semi open set.*
3. *A is contained in $si_\mu(A \cup (X - sc_\mu(A)))$.*

Proof. 1.-Suppose that A is locally μ-semi regular semi closed subset of X, then there exists a semi regular open set U such that $A = U \cap sc_\mu(A)$. Follows that: $sc_\mu(A) - A = sc_\mu(A) - (U \cap sc_\mu(A)) = sc_\mu(A) \cap (X - (U \cap sc_\mu(A))) = sc_\mu(A) \cap ((X - U) \cup (X - sc_\mu(A))) = sc_\mu(A) \cup (X - U) \cap sc_\mu(A) \cap (X - sc_\mu(A)) = sc_\mu(A) \cap (X - U)$. Now $sc_\mu(A)$ is μ-semi closed and $X - U$ is semi regular closed and $SRO(X) \subset \mu$, we obtain that $X - U$ and then $sc_\mu(A) \cap (X - U)$ is μ-semi closed.
2.- Using (1), $sc_\mu(A) - A$ is μ-semi closed, then its complement $X - (sc_\mu(A) - A)$ is μ-semi open, but $X - (sc_\mu(A) - A) = X - (sc_\mu(A) \cap (X - A) = A \cup (X - sc_\mu(A))$.
3.-Using (2), $A \subset (A \cup (X - sc_\mu(A))) = si_\mu(A \cup (X - sc_\mu(A)))$. □

Remark 2.9. The next example shows that $SRO(X) \subset \mu$ is necessary in the above theorem.

Example 2.5. Let $X = \{a, b, c\}$ with topology $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$ and GT $\mu = \{\emptyset, \{b, c\}\}$. Observe that:
$SRO(X) = \{\emptyset, X, \{a\}, \{b\}, \{a, c\}, \{b, c\}\}$. If we take $A = \{c\}$, $sc_\mu(A) = \{b, c\}$ and $sc_\mu(A) - A = \{b\}$ is not μ-semi closed.

At this point there are a question, there exist any relation between the locally μ-semi closed set and the locally μ-semi regular semi closed set. We know that between the open sets and the semi regular open sets there are no relations, in this case, we affirm that both concepts are independent as is shown in the next examples.

Example 2.6. Let $X = \{a, b, c\}$ with topology $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$ and GT $\mu = \{\emptyset, \{b, c\}\}$.

1. The set of locally μ-sclosed sets $=\{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}\}$.
2. The set of locally μ-srsclosed sets $=\{\emptyset, X, \{a\}, \{b\}, \{c\}, \{a, c\}, \{b, c\}\}$.

Observe that $\{a, b\}$ is locally μ-semi closed set but is not locally μ-srsclosed set in the same form $\{a, c\}$ locally μ-srsclosed set but is not locally semi μ-semi closed.

Definition 2.6. Let μ be a GT on a topological space (X, τ). A subset A of X is called:

1. $\widehat{\mu}$-t-set if $int(cl(A)) = int(cl(c_\mu(A)))$.
2. $\widehat{\mu}$-B-set if $A = U \cap V$, $U \in RO(X)$, V is a μ-t-set.
3. $\mu^{,,}$-open set if $A \subseteq int(cl(c_\mu(A)))$.

Remark 2.10. If μ is a GT on a a topological space (X, τ), then

1. If A is a μ-closed set then it is a $\widehat{\mu}$-t-set.
2. If A is a $\widehat{\mu}$-t-set then it is a $\widehat{\mu}$-B-set.
3. Every locally μ-regular closed set is a $\widehat{\mu}$-B-set.

We point out that in the article [5], Proposition 2.3, which claims that if μ is a GT on a topological space (X, τ). Then A is regular open if and only if A is $\widehat{\mu}$-B-set and $\mu^{,,}$-open set. Is false, as we can see from the following example.

Example 2.7. Let $X = \{a, b, c, d\}$ with topology $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$, Consider the GT $\mu = \{\emptyset, \{a\}, \{a, b\}, \{a, b, c\}\}$. Observe that:

1. $RO(X) = \{\emptyset, X, \{a\}, \{b\}\}$.
2. $\widehat{\mu}$-t-set $=\{\emptyset, X, \{b\}, \{c\}, \{d\}, \{a, b\}, \{b, c\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\}\}$.
3. $\widehat{\mu}$-B-set $=\{\emptyset, X, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{b, c\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\}\}$.
4. $\mu^{,,}$-open set $=\{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}\}$.

Observe that $\{a, b, c\}, \{a, b, d\}$ both are $\widehat{\mu}$-B-set and $\mu^{,,}$-open set but not regular open.

Definition 2.7. Let μ be a GT on a topological space (X, τ). A subset A of X is called:

1. $\widehat{\mu}$-st-set if $int(cl(A)) = int(cl(sc_\mu(A)))$.
2. $\widehat{\mu}$-sB-set if $A = U \cap V$, $U \in RO(X)$, V is a μ-st-set.
3. $\mu^{,,}$-sopen set if $A \subseteq int(cl(sc_\mu(A)))$.
4. $\widehat{\mu}$-$\widehat{sB}$-set if $A = U \cap V$, $U \in SRO(X)$, V is a μ-st-set.

Theorem 2.7. *If μ is a GT on a topological space (X, τ), then*

1. *If A is a μ-semiclosed set then it is a $\widehat{\mu}$-st-set.*
2. *If A is a $\widehat{\mu}$-st-set then it is a $\widehat{\mu}$-sB-set.*
3. *Every locally μ-regular sclosed set is a $\widehat{\mu}$-sB-set.*
4. *If A is a $\widehat{\mu}$-sB-set then it is a $\widehat{\mu}$-$\widehat{sB}$-set.*

Proof. 1. Let A be a μ-semi closed set, then $A = sc_\mu(A)$. It follows that $int(cl(A)) = int(cl(sc_\mu(A)))$, in consequence, A is $\widehat{\mu}$-st-set.
2. Suppose that A is a $\widehat{\mu}$-st-set. Since X is a regular open set and $A = A \cap X$, A is a $\widehat{\mu}$-sB-set.
3. It follows from Definition 2.3 and 2.
4. Since every regular open set is semi regular open, the result follows. □

The following examples shows that the converse of Theorem 2.7 not necessarily is true.

Example 2.8. *Let $X = \{a, b, c\}$ with topology $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$ and GT $\mu = \{\emptyset, \{b, c\}\}$.*

1. *Regular open sets* $=\{\emptyset, X, \{a\}, \{b\}\}$.
2. *μ-semi open sets* $= \{\emptyset, X, \{a\}, \{b, c\}\}$.
3. *μ-semi closed sets* $= \{\emptyset, X, \{a\}, \{b, c\}\}$.
4. *Semi regular open sets* $= \{\emptyset, X, \{a\}, \{b\}, \{a, c\}, \{b, c\}\}$.
5. *Locally μ rclosed* $= \{\emptyset, X, \{a\}, \{b, \}\}$.
6. *Locally μ rsclosed* $= \{\emptyset, X, \{a\}, \{b\}, \{b, c\}\}$.
7. *Locally μ srsclosed* $=\{\emptyset, X, \{a\}, \{b\}, \{c\}, \{a, c\}, \{b, c\}\}$.
8. *$\widehat{\mu}$-st-sets*$= \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}\}$.
9. *$\widehat{\mu}$-sB-sets* $=\{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{b, c\}\}$.
10. *$\widehat{\mu}$-$\widehat{sB}$-sets* $=\{\emptyset, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$.
11. *$\mu^{''}$-sopen sets* $=\{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}\}$.

Observe that $\{a, b\}$ is $\widehat{\mu}$-st-set but is not μ-semi closed.
$\{a, c\}$ is $\widehat{\mu}$-$\widehat{sB}$-set but is not $\widehat{\mu}$-st-set.
$\{a, b\}$ is $\widehat{\mu}$-sB-set but is not locally μ-regular semi closed.

Example 2.9. Let $X = \{a, b, c, d\}$ with topology $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$ and GT. $\mu = \{\emptyset, \{a\}, \{a, b, c\}\}$.

1. Regular open sets $=\{\emptyset, X, \{a\}, \{b\}, \{c\}, \{d\}\}$.
2. $\widehat{\mu}$-st-sets$= \{\emptyset, X, \{b\}, \{d\}, \{a, b\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}\}$.
3. $\widehat{\mu}$-sB-sets $=\{\emptyset, X, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}\}$.

Observe that $\{a\}$ is a $\widehat{\mu}$-sB-set but is not $\widehat{\mu}$-st-set.

3. Generalized μ-semiregular semiclosed sets

Definition 3.8. [1] Let μ be a GT on a topological space (X, τ). A subset A of X is called generalized μ-semi closed (briefly $g\mu$-sclosed) if $sc_\mu(A) \subseteq U$ where $A \subseteq U$ and $U \in \tau$.

Definition 3.9. Let μ be a GT on a topological space (X, τ). A subset A of X is called generalized μ-regular closed (briefly $g\mu$-rclosed) if $c_\mu(A) \subseteq U$ where $A \subseteq U$ and $U \in RO(X)$.

Definition 3.10. Let μ be a GT on a topological space (X, τ). A subset A of X is called generalized μ-regular semiclosed (briefly $g\mu$-rsclosed) if $sc_\mu(A) \subseteq U$ where $A \subseteq U$ and $U \in RO(X)$.

Definition 3.11. Let μ be a GT on a topological space (X, τ). A subset A of X is called generalized μ-semiregular closed (briefly $g\mu$-srclosed) if $c_\mu(A) \subseteq U$ where $A \subseteq U$ and $U \in SRO(X)$.

Definition 3.12. Let μ be a GT on a topological space (X, τ). A subset A of X is called generalized μ-semiregular semiclosed (briefly $g\mu$-srsclosed) if $sc_\mu(A) \subseteq U$ where $A \subseteq U$ and $U \in SRO(X)$.

Remark 3.11. It is easy to see that:

1. $g\mu$-srclosed$\Rightarrow g\mu$-sclosed$\Rightarrow g\mu$-rsclosed.

2. $g\mu$-srclosed$\Rightarrow g\mu$-srsclosed$\Rightarrow g\mu$-rsclosed.

3. $g\mu$-srclosed$\Rightarrow g\mu$-rclosed$\Rightarrow g\mu$-rsclosed.

But the converse are not necessarily true, as we can see in the following examples.

Example 3.10. Let $X = \{a, b, c\}$ with topology $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$ and GT $\mu = \{\emptyset, \{b, c\}\}$.

1. $g\mu$-sclosed $=\{\emptyset, X, \{a\}, \{c\}, \{a, c\}, \{b, c\}\}$.
2. $g\mu$-rclosed $=\{\emptyset, X, \{a\}, \{c\}, \{a, b\}\{a, c\}, \{b, c\}\}$.
3. $g\mu$-rsclosed $=\{\emptyset, X, \{a\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$.
4. $g\mu$-srclosed $=\{X, \{a\}\}$.
5. $g\mu$-srsclosed $=\{\emptyset, X, \{a\}, \{a, b\}, \{b, c\}\}$.

Example 3.11. Let $X = \{a, b, c\}$ with topology $\tau = \{\emptyset, X, \{a\}, \{c\}, \{a, c\}\}$ and GT $\mu = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$.

1. $g\mu$-rclosed $=\{\emptyset, X, \{c\}, \{a, c\}\{a, c\}, \{b, c\}\}$.
2. $g\mu$-rsclosed $=\{\emptyset, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$.

Theorem 3.8. *Let μ be a GT on a topological space (X, τ). $A \subset X$ is μ-closed if and only if A is $g\mu$-rclosed and locally μ-closed.*

Proof. Suppose that A is μ-closed and $A \subset U$ where $U \in RO(X)$. Since $A = c_\mu(A)$, we obtain that A is $g\mu$-rclosed and locally μ-rclosed.
Conversely. Suppose that A is $g\mu$-rclosed and locally μ- closed, then $A = U \cap F$, where $U \in RO(X)$ and F is μ-closed, therefore, $A \subset U$ and $A \subset F$, in consequence, $c_\mu(A) \subset U$ and $c_\mu(A) \subset F$. Follows that $c_\mu(A) \subset U \cap F = A$. So A is μ-closed. □

Theorem 3.9. *Let μ be a GT on a topological space (X, τ). $A \subset X$ is μ-closed if and only if A is $g\mu$-rclosed and locally μ-rclosed.*

Proof. Suppose that A is μ-closed and $A \subset U$ where $U \in RO(X)$. Since $A = c_\mu(A)$, we obtain that A is $g\mu$-rclosed and locally μ-rclosed.
Conversely. Suppose that A is $g\mu$-rclosed and locally μ- closed, then $A = U \cap F$, where $U \in RO(X)$ and F is μ-closed, therefore, $A \subset U$ and $A \subset F$, in consequence, $c_\mu(A) \subset U$ and $c_\mu(A) \subset F$. Follows that $c_\mu(A) \subset U \cap F = A$. So A is μ-closed. □

Theorem 3.10. *Let μ be a GT on a topological space (X, τ). $A \subset X$ is μ-semiclosed if and only if A is $g\mu$-rsclosed and locally μ-semiclosed.*

Proof. Suppose that A is μ-semiclosed and $A \subset U$ where $U \in RO(X)$. Since $A = sc_\mu(A)$, we obtain that A is $g\mu$-rsclosed and locally μ-srclosed.
Conversely. Suppose that A is $g\mu$-rsclosed and locally μ- srclosed, then $A = U \cap F$, where $U \in RO(X)$ and F is μ-semiclosed, therefore, $A \subset U$ and $A \subset F$, in consequence, $sc_\mu(A) \subset U$ and $sc_\mu(A) \subset F$. Follows that $sc_\mu(A) \subset U \cap F = A$. So A is μ-semiclosed. □

Theorem 3.11. *Let μ be a GT on a topological space (X, τ). $A \subset X$ is μ-semiclosed if and only if A is $g\mu$-srsclosed and locally μ-semi regular semiclosed.*

Proof. Suppose that A is μ-semiclosed and $A \subset U$ where U in $SRO(X)$. Since $A = sc_\mu(A)$, we obtain that A is $g\mu$-srsclosed and locally μ-semiregular semiclosed.
Conversely. Suppose that A is $g\mu$-srsclosed and locally μ- semiregular semiclosed, then $A = U \cap F$, where $U \in SRO(X)$ and F is μ-semiclosed, therefore, $A \subset U$ and $A \subset F$, in consequence, $sc_\mu(A) \subset U$ and $sc_\mu(A) \subset F$. Follows that $sc_\mu(A) \subset U \cap F = A$. So A is μ-semiclosed. □

4. (μ,σ)-RCONTINUOUS FUNCTIONS

Definition 4.13. Let μ be a GT on a topological space (X,τ). A function $f:(X,\tau)\to(Y,\sigma)$ is (μ,σ)-rscontinuous if $f^{-1}(V)$ is μ-semi open in X for each regular open set V of Y.

Observe that every (μ,σ)-scontinuous function ([6]) is (μ,σ)-rscontinuous function, but the converse is false.

Definition 4.14. Let μ be a GT on a topological space (X,τ). A function $f:(X,\tau)\to(Y,\sigma)$ is (μ,σ)-srscontinuous if $f^{-1}(V)$ is μ-semi open in X for each semi regular open set V of Y.

Observe that every (μ,σ)-srscontinuous function is (μ,σ)-rscontinuous function, but the converse is false.

Definition 4.15. Let μ be a GT on a topological space (X,τ). Then $f:(X,\tau)\to(Y,\sigma)$ is said to be gμ-regular semi continuous(briefly gμ-rscontinuous) (respectively contra locally gμ-regular semi continuous (briefly contra locally gμ-rscontinuous)) if $f^{-1}(F)$ is a gμ-rsclosed (respectively locally μ-rsclosed) for each regular closed set F of (Y,σ).

Definition 4.16. Let μ be a GT on a topological space (X,τ). Then $f:(X,\tau)\to(Y,\sigma)$ is said to be gμ-semi regular semi closed (briefly gμ-srscontinuous) (respectively contra locally gμ-semi regular semi continuous(briefly contra locally gμ-srscontinuous)) if $f^{-1}(F)$ is a gμ-srsclosed (respectively locally μ-srsclosed) for each semi regular closed set F of (Y,σ).

Theorem 4.12. *Let μ be a GT on a topological space (X,τ). Then for a function $f:(X,\tau)\to(Y,\sigma)$, the following are equivalent:*

1. *f is (μ,σ)-rscontinuous.*
2. *$f^{-1}(F)$ is μ-semi closed for each semi regular closed set F of (Y,σ).*

Proof. The proof is clear. □

Theorem 4.13. *Let μ be a GT on a topological space (X,τ). Then for a function $f:(X,\tau)\to(Y,\sigma)$, the following are equivalent:*

1. *f is (μ,σ)-srscontinuous.*
2. *$f^{-1}(F)$ is μ-semi closed for each semi regular closed set F of (Y,σ).*

Proof. The proof is clear. □

Theorem 4.14. *Let μ be a GT on a topological space (X,τ). Then $f:(X,\tau)\to(Y,\sigma)$ is (μ,σ)-rscontinuous if and only if it is gμ-rscontinuous and contra locally gμ-rscontinuous.*

Proof. The proof follows from Theorem 3.11 and Theorem 4.13. □

In analogous form, we can proof the following theorem.

Theorem 4.15. *Let μ be a GT on a topological space (X,τ). Then $f:(X,\tau)\to(Y,\sigma)$ is (μ,σ)-rscontinuous if and only if it is gμ-rscontinuous and contra locally μ-rscontinuous.*

We point out again in the next example that the result given in Theorem 2.8 of [6] is false. First, we give two definitions, the theorem and the example.

Definition 4.17. [6] *Let μ be a GT on a topological space (X,τ). A mapping $f:(X,\tau)\to(Y,\sigma)$ is said to be μ''-continuous (resp. $\widehat{\mu}$-B-continuous) if $f^{-1}(F)$ is a μ''-open (resp. $\widehat{\mu}$-B-set) for each regular open set F of Y.*

Definition 4.18. [6] Let μ be a GT on a topological space (X,τ). A mapping $f:(X,\tau)\to(Y,\sigma)$ is R-map if $f^{-1}(F)\in RO(X)$.

Now, we write down the Theorem 2.8, given in [6].

Theorem 4.16. *Let μ be a GT on a topological space (X, τ). A mapping $f : (X, \tau) \to (Y, \sigma)$ is an R-map if and only if it is $\mu^{"}$-continuous and $\widehat{\mu}$-B-continuous.*

Example 4.12. Let $X = Y\{a, b, c\}$, $\tau = \sigma = \{\emptyset, X, \{a\}, \{c\}, \{a, c\}, \{b, c\}\}$ and GT $\mu = \{\emptyset, \{b, c\}\}$

1. $RO(X) = \{\emptyset, X, \{a\}, \{b, c\}\}$.
2. $\widehat{\mu} - t - set = \{\emptyset, X, \{a\}, \{a, c\}\}$.
3. $\widehat{\mu} - B - set = \{\emptyset, X, \{a\}, \{c\}, \{a, c\}, \{b, c\}\}$.
4. $\mu^{"} - open = \{\emptyset, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$.

Define $f : (X, \tau) \to (Y, \sigma)$ as follows: $f(a) = c, f(b) = a$ and $f(c) = b$. Observe that: $f^{-1}(\{a\}) = \{b\}, f^{-1}(\{b, c\}) = \{a, c\}$. f is both $\mu^{"}$-continuous and $\widehat{\mu}$-B-continuous but is not an R-map.

REFERENCES

[1] Carpintero, C., Moreno, J. and Rosas, E., *Some New type of decomposition of continuity*, submitted.

[2] Császár, Á., *Generalized topology, generalized continuity*, Acta Math. Hungar., **96** (2002), 351–357

[3] Császár, Á., *Generalized open sets in generalized topologies*, Acta Math. Hungar., **96** (2005), 53–66

[4] Császár, Á., *δ and θ-modifications of generalized topologies*, Acta Math. Hungar., **120** (2008), No. 3, 275–279

[5] Roy, B. and Sen, R., *On a type of decomposition of continuity*, Afr. Mat., **26** (2015), No. 1-2, 153–158

[6] Roy, B. and Sen, R., *On decomposition of weak continuity*, Creat. Math. Inform., **24** (2015), No. 1-2, 83–88

[7] Rosas, E. Rajesh, N. and Carpintero, C., *Some New Types of Open and Closed Sets in minimal Structure part I*, Int. Math. Forum., **4** (2009), No. 44, 2169–2184

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