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Dedicated to Professor Iulian Coroian on the occasion of his 70^{th} anniversary

A problem of entropy generation in a channel filled with a porous medium

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ABSTRACT. The problem studied is that of entropy generation for mixed convection in an inclined channel. The channel is filled with a porous medium and has an uniform wall heat flux. The flow is upward and the heat flux is into the channel. The solutions of the governing Darcy and energy equations are used for analyzing the entropy generation and the Bejan number into the channel. The results are plotted and studied for different important parameters involved and for different inclinations angle of the channel.

1. INTRODUCTION

Minimization of entropy generation is a method for modeling and optimizing of energy systems (see Bejan [3]) which results from the analysis of the second law of thermodynamics. In earlier studies related to the natural convection, only the first-law of thermodynamics was used. However, the method of entropy generation combines from the start the most important parameters of thermodynamics, heat transfer and fluid mechanics. To improve the heat transfer performance is a chief task in heat exchanger designs (see Ingham and Pop [5]). Owing to the fact that the heat transfer enhancement is always achieved at the expense of the increase of friction loss, the optimal trade-off by selecting the most appropriate configuration and the best flow conditions has become the critical challenge for the design work. The analysis of the energy utilization and the entropy generation has become one of the primary objectives in designing a thermal system. Bejan [2], has described the systematic methodology of computing entropy generation through heat and fluid flow in heat exchangers. Fundamentals of entropy generation are also presented by Rosen [7] and Narusawa [6].

The aim of this study is to present results for entropy generation due to mixed convection heat transfer in an inclined channel filled with a porous medium, for different parameters involved, as Rayleigh numbers or the inclination angle of the channel. The Bejan number is also presented and the results are observed.

2. MATHEMATICAL MODELING

Consider the mixed convection flow between two inclined parallel plates filled with a porous medium, see Figure 1. The x axis is considered up lengthways and

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the y axis is oriented into the channel. The flow is assumed to be fully developed (one-dimensional) and steady, and the fluid and porous media properties are constant except for the variation of density in the buoyancy term of the Darcy equation. The porous medium is considered to be homogeneous and isotropic. Also, the fluid within the porous medium is assumed to saturate the solid matrix and both are in local thermodynamic equilibrium. The fluid has an uniform upward (assisting flow) streamwise velocity distribution at the channel entrance. The walls are at uniform heat flux q. Under these assumptions, and with the use of the Darcy's law and the Boussinesq approximation, the governing equations are written as follows

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2.1}$$

$$\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} = \frac{g\beta K}{\nu}\left(\frac{\partial T}{\partial y}\sin\gamma - \frac{\partial T}{\partial x}\cos\gamma\right) \tag{2.2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha_m\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) \tag{2.3}$$

where u and v are the cartesian velocity components, T is the fluid temperature. The coefficients are β the fluid thermal expansion, K the specific permeability of the medium, ν the kinematic viscosity and α_m the effective fluid thermal diffusivity. Also, the tilt angle, measured counterclockwise from the horizontal is denoted by γ in the considered equations.

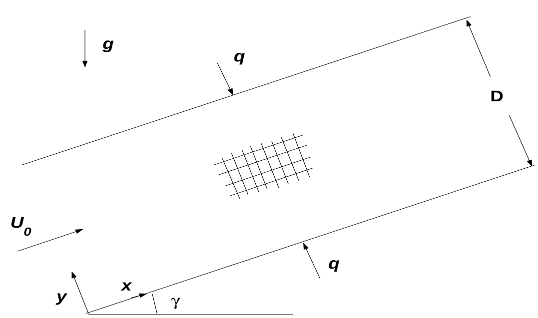

Figure 1. The parallel plates configuration for upward flow.

The assumption of fully development flow means that the axial (x direction) velocity u depends only on the transverse coordinate y, i.e. $u = u(y)$. Then, from Eq. (2.1) we have $\frac{\partial v}{\partial y} = 0$ which, due to the boundary condition $v = 0$ on $y = 0$ gives $v \equiv 0$. Also the boundary conditions are:

$$\frac{\partial T}{\partial y} = -\frac{q}{k} \text{ on } y = 0; \ \frac{\partial T}{\partial y} = \frac{q}{k} \text{ on } y = D \tag{2.4}$$

where q is the heat flux to the wall, D is the channel width and k is the magnitude of the fluid thermal conductivity of the porous medium.

We consider that the temperature T is an arbitrary function of y and a linear function of x and we introduce the following non-dimensional variables:

$$X = \frac{\alpha_m}{U_0 D^2} x, \quad Y = \frac{y}{D}, \quad U = \frac{u}{U_0}, \quad \theta = \frac{T - T_0}{qD/k} \tag{2.5}$$

Then we have $\theta(X,Y) = C_1 X + F(Y)$ and following the paper by Cimpean *et al.* [4], a third order ordinary differential equation is obtained:

$$\frac{d^3F}{dY^3} - 2P_1 \frac{dF}{dY} + 4P_2 = 0 \tag{2.6}$$

which has to be solved for $\frac{dF}{dY}$, subject to the boundary conditions:

$$\frac{dF}{dY} = -1 \text{ at } Y = 0; \ \frac{dF}{dY} = 1 \text{ at } Y = 1 \tag{2.7}$$

In the Eq. (2.6) the parameters are $P_1 = \frac{Ra}{Pe} \sin\gamma$ and $P_2 = \frac{Ra}{Pe^2} \cos\gamma$, where $Ra = \frac{g\rho\beta K(q_w D/k)D}{\alpha_m \mu}$ and $Pe = \frac{U_0 D}{\alpha_m}$ are the $Rayleigh$ and $Péclet$ number, respectively.

The analytical solutions for the velocity and temperature profiles are:

$$U = \frac{\sqrt{2P_1}}{2} \left[C_2 e^{\sqrt{2P_1 Y}} - C_3 e^{-\sqrt{2P_1 Y}} \right] \tag{2.8}$$

$$\theta(X,Y) = \frac{1}{\sqrt{2P_1}} \left(C_2 e^{\sqrt{2P_1 Y}} - C_3 e^{-\sqrt{2P_1 Y}} \right) + 2\frac{P_2}{P_1} Y + 2X + C_4 \tag{2.9}$$

3. Entropy generation

The entropy generation is caused by the non-equilibrium state of the fluid, resulting from the thermal gradient between the two media. For the problem involved, the exchange of energy and momentum within the fluid-saturated porous medium and at the solid boundaries, give the non-equilibrium conditions which cause the entropy generation in the flow field of the channel. This entropy generation is due to the irreversible nature of heat transfer and viscosity effects, within the fluid and at the solid boundaries. From the known temperature and velocity fields, volumetric entropy generation can be calculated by the equation (see Bejan [2] and Baytas [1]):

$$S_{gen}^m = \frac{k}{T_0^2} (\nabla T)^2 + \frac{\mu}{KT_0} \left(u^2 + v^2 \right) \tag{3.10}$$

Further, we transform the above Eq. (3.10) into the dimensionless form by using the expressions (2.5) and we will obtain the dimensionless entropy generation number, N_s, as follows:

$$N_s = \frac{4}{Pe^2} + \left(\frac{\partial\theta}{\partial Y} \right)^2 + \Phi U^2 \tag{3.11}$$

where Pe is the Peclet number and Φ is called the irreversibility distribution ratio (see Baytas [1]), given as $\Phi = \frac{\mu T_0}{k} \left[\frac{\alpha_m^2}{K(\Delta T)^2} \right]$.

The total local entropy generation number can be written as a summation of the local entropy generation due to heat transfer (HTI) and the local entropy generation due to fluid friction (FFI), as $N_s = HTI + FFI$. The last expression gives us the possibility to calculate these terms separately and then compare them to notice which entropy generation mechanism dominates. In the convection problems, both, fluid friction and heat transfer, contribute to the rate of entropy generation. The entropy in a system is associated with the presence of irreversibility. We have to notice that the contribution of the heat transfer entropy generation, HTI, to the overall entropy generation rate, is needed in many engineering applications.

As it is well known, the Bejan number, (Be), is an alternative irreversibility distribution parameter and represents the ratio between the heat transfer irreversibility (HTI) and the total irreversibility due to heat transfer and fluid friction (N_s). It is defined by $Be = \frac{HTI}{N_s}$ and takes the values between 0 and 1. The value of $Be = 1$ is the limit at which the heat transfer irreversibility dominates, $Be = 0$ is the opposite limit at which the irreversibility is dominated by fluid friction effects and $Be = 0.5$ is the case in which the heat transfer and fluid friction entropy production rates are equal (Varol et al. [8], [9]).

4. Results and discussion

The fluid has an uniform, upward, streamwise velocity distribution at the channel entrance in the same direction with the convective flow. Figure 2 express the entropy generation number as a function of Y, for different values of Φ, the irreversibility distribution ratio, for $P_2 = 0$ (vertical channel), $P_2 = 10$, and $P_2 = -10$. The entropy generation number is significantly higher at the walls (see Figure 2a) and decreases to a minimum value to the middle of the channel (see Figure 2b). By increasing the parameter Φ, the entropy generation number increases. For vertical channel ($P_2 = 0$), a similar behavior is observed on the walls. Also the N_s profiles are similar on the lower wall for $P_2 = -10$ to profiles on the upper wall for $P_2 = 10$ and vice versa.

Figure 3 plot the entropy generation number and fluid friction irreversibility as functions of the inclination angle of the channel, γ. It is shown that, near the walls, the entropy generation number and also FFI have a symmetric behavior about the value $\gamma = \pi/2$. By increasing the parameter Φ, the values of N_s and FFI increase. Fluid friction irreversibility values are smaller that the values of entropy generation number. Near by lower wall, the values of entropy generation number increases and the fluid friction irreversibility decreases with increasing the inclination angle of the channel (see Figure 3a). A vice versa phenomenon is observed in the vicinity of the upper wall, as can be observed in the Figure 3b.

Figure 4 represents the behavior of both, entropy generation number and heat transfer irreversibility, HTI, versus the inclination angle of the channel. By comparing this figure to Figure 3, we conclude that the inclination angle of the channel has a much important role on HTI than on FFI.

Figure 5 shows the Bejan number versus Rayleigh number, for values of the parameter $\Phi = 10^{-1}, 10^{-2}, 10^{-3}$ and the values of the inclination angle of the channel $\gamma = \pi/6, \pi/4, \pi/3$. For all parameters involved, as increasing Rayleigh, the Bejan number yields to a constant value. It is important to observe that for $\Phi = 10^{-2}$, FFI dominates into a small Rayleigh interval ($Be < 0.5$) and HTI dominates for the rest of the Rayleigh values ($Be > 0.5$). Also, higher values of the Bejan number are obtained for lower values of the inclined angle of the channel.

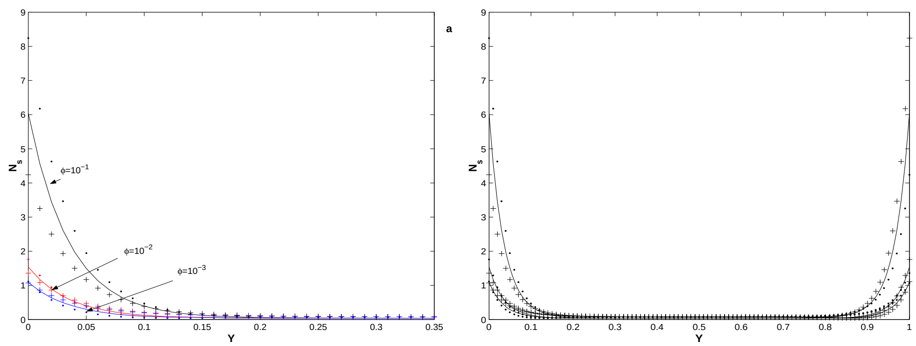

Figure 2. Entropy generation profiles, N_s, as functions of Y, for $P_2 = 0$ (vertical channel), plotted by line, $P_2 = -10$, plotted by plus signs and $P_2 = 10$, shown by dotes.

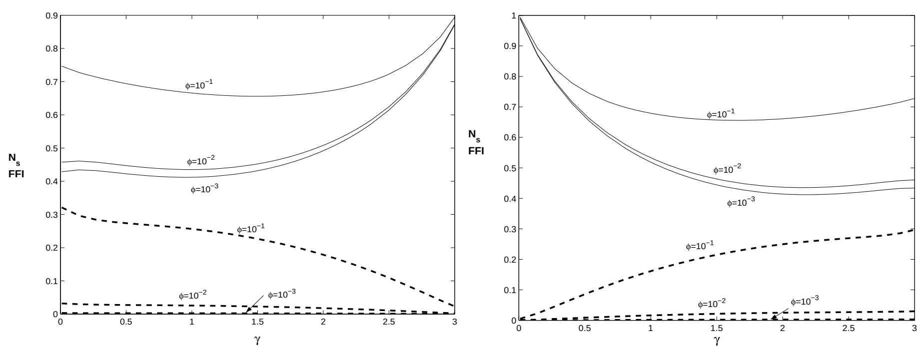

Figure 3. Entropy generation number N_s (shown by line) and FFI (shown by broken line) versus inclined angle γ., in the vicinity of the lower wall (a) and upper wall (b).

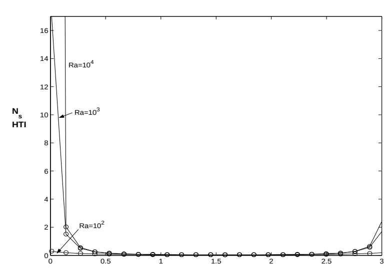

Figure 4. Entropy generation number N_s (shown by line) and HTI (shown by circles) versus inclined angle γ., for different Ra numbers).

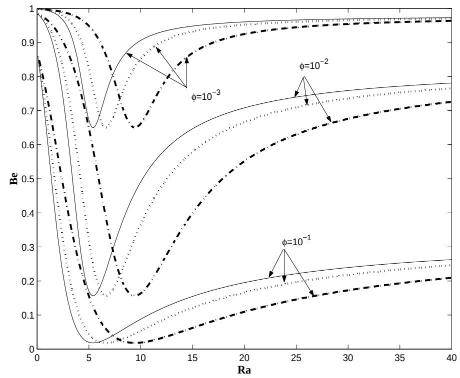

Figure 5. Bejan number Be versus Ra number, for different Φ parameters and for values of the inclination angle $\gamma = \pi/6$ (shown by line), $\gamma = \pi/4$ (shown by dotted) and $\gamma = \pi/3$ (shown by broken line).

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